Lecture 8
Continuous-Wave Laser*

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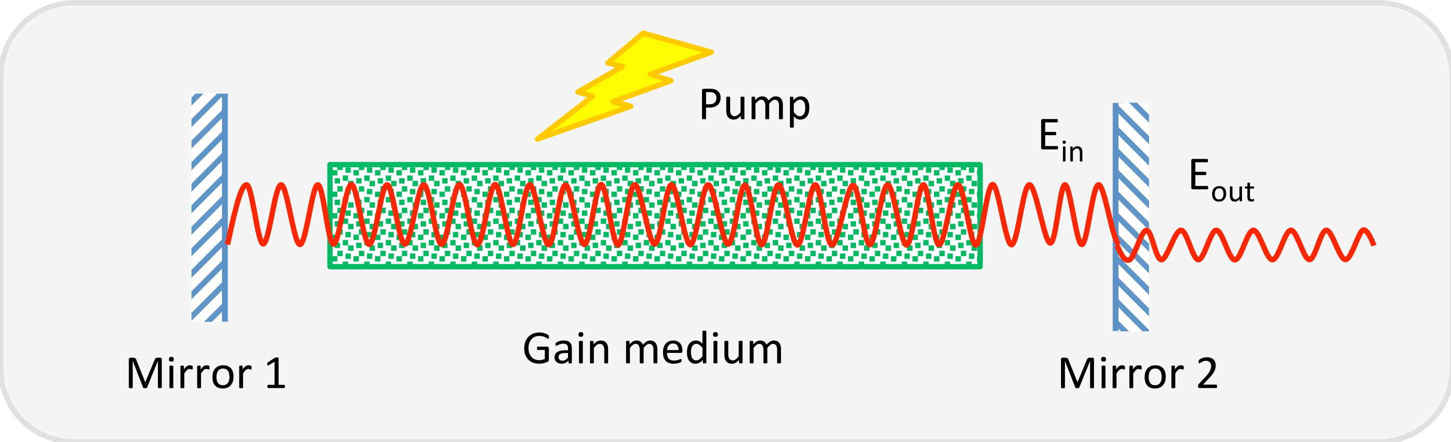
* Some figures and texts belong to: O. Svelto, *Principles of Lasers*, 5th Ed., Springer.

Reading

- *Principles of Lasers* (5th Ed.): Chapter 7.
- Skip: 7.3.2, 7.4.2, 7.8.2.2.
- Squeeze: 7.9, 7.10.

Laser



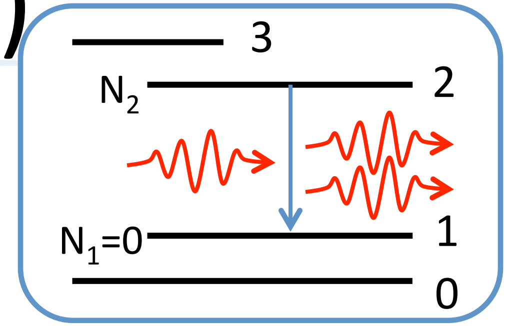
- Rate equation (interplay between N and ϕ)
- Threshold conditions
- Steady-state N, ϕ, P_{out}, η_s
- R_2 for optimum P_{out}
- Single-mode selection, and tuning

Contents

Content	Time
1. Rate equations 1. Four-level; 2. Quasi-three-level	25’
2. Threshold and steady states 1. Four-level; 2. Quasi-three-level	15’
3. Optimum output coupling	5’
4. Single-mode selection and tuning	30’
5. Others Frequency-pulling, fluctuations	5’
Total:	80’

Contents

Rate equations (4L)

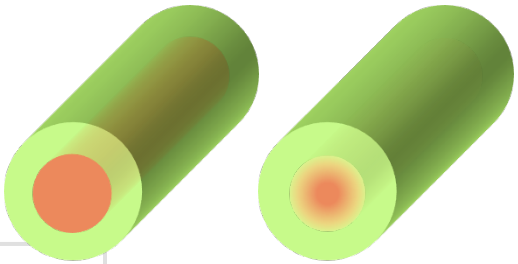


Stimulated emission

$$\frac{dN_2}{dt} = R_p - \boxed{B\phi N_2} - \frac{N_2}{\tau}$$

$$\frac{d\phi}{dt} = \boxed{V_a B\phi N_2} - \frac{\phi}{\tau_c}$$

Stimulated emission



Assumptions:
- Single-mode
- Pump & laser mode are uniform (space-independent)

- N_2: Population inversion (per unit volume)
- ϕ: Total photon number
- B: Stimulated transition rate per photon per mode
- τ: Effective upper-level lifetime [radiative (Spon.E.)+nonradiative]
- V_a: Volume of the mode in the active region
- τ_c: Cavity photon lifetime

B

Laser intensity change after one round trip

$$\Delta I = I \cdot R_1 R_2 (1 - L_i)^2 \cdot \exp(2\sigma N_2 l) - I$$

Define single-trip logarithmic loss as

$$\gamma = -\frac{1}{2} \ln \left[R_1 R_2 (1 - L_i)^2 \right]$$

After round trip, ΔI becomes

$$\Delta I = I \cdot \exp\left[2(\sigma N_2 l - \gamma)\right] - I$$

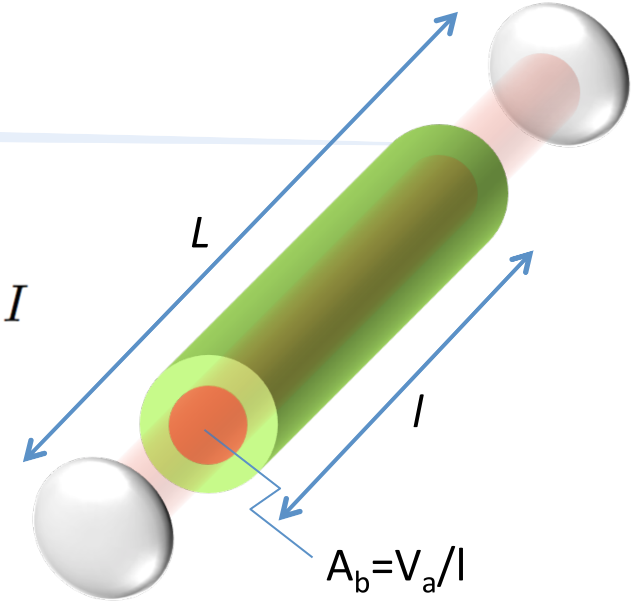
If $\sigma N_2 l - \gamma \ll 1$

$$\Delta I = 2I(\sigma N_2 l - \gamma)$$

Divide by Δt (round-trip time)

$$\frac{dI}{dt} = \frac{\sigma l c}{L_e} N_2 I - \frac{\gamma c}{L_e} I$$

$$\boxed{\frac{d\phi}{dt} = V_a B \phi N_2 - \frac{\phi}{\tau_c}}$$



Round-trip time: $\Delta t = 2L_e/c$, where $L_e = L + (n-1)l$

Since $I \propto \phi$, by comparison with 2nd rate eq.

$$\boxed{B = \frac{\sigma l c}{V_a L_e} = \frac{\sigma c}{V} \qquad \tau_c = \frac{L_e}{\gamma c}}$$

V: cavity mode volume

Rate equations…so what?

$$\boxed{\begin{aligned}\frac{dN_2}{dt} &= R_p - B\phi N_2 - \frac{N_2}{\tau}\\ \frac{d\phi}{dt} &= V_a B\phi N_2 - \frac{\phi}{\tau_c}\end{aligned}}$$

1. CW characteristics
   - Threshold-state condition: $\phi \approx 0$, N_c
   - Steady-state condition: $d\phi/dt=0$, $dN/dt=0$
2. Transient characteristics
   - $\phi(t)$ and $N(t)$ can be derived if $\phi(t=0)$ and $R_p(t)$ are given
3. Output power P_{out} if $\phi(t)$ is known
4. Slope efficiency η_s, i.e. dP_{out}/dP_p

To get P_{out}: $I = I_0 \exp\left(-\frac{t}{\tau_c}\right) = I_0 \exp\left(-\frac{\gamma c t}{L_e}\right) = I_0 \exp\left[-\frac{(\gamma_1+\gamma_2+2\gamma_i)ct}{2L_e}\right]$

$$\boxed{\frac{1}{\tau_c} = \frac{\gamma_1}{2L_e} + \frac{\gamma_2}{2L_e} + \frac{\gamma_i}{L_e}} \rightarrow \boxed{\left.\frac{dI}{dt}\right|_{\gamma_2} = -\frac{\gamma_2 c}{2L_e} I} \rightarrow \boxed{P_{out} = \phi \frac{\gamma_2 c}{2L_e} h\nu}$$

Rate equations (q3L)

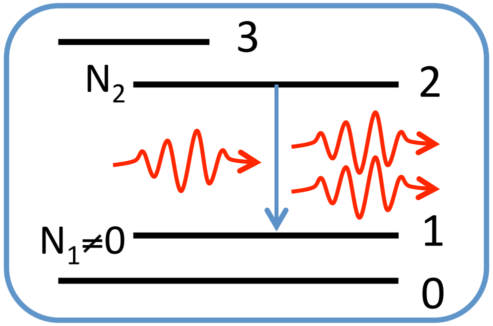
$$N_1 + N_2 = N_t$$

$$\frac{dN_2}{dt} = R_p - \phi(B_e N_2 - B_a N_1) - \frac{N_2}{\tau}$$

$$\frac{d\phi}{dt} = V_a \phi(B_e N_2 - B_a N_1) - \frac{\phi}{\tau_c}$$

$$B_e = \frac{\sigma_e c}{V}$$

$$B_a = \frac{\sigma_a c}{V}$$



If we define $f=\sigma_a/\sigma_e$ and population inversion $N=N_2-fN_1$

$$\frac{dN}{dt} = R_p(1+f) - \frac{(\sigma_e+\sigma_a)c}{V}\phi N - \frac{fN_t + N}{\tau}$$

$$\frac{d\phi}{dt} = \frac{V_a \sigma_e c}{V} N\phi - \frac{\phi}{\tau_c}$$

4L-case

$$\frac{dN}{dt} = R_p - \frac{\sigma c}{V}\phi N - \frac{N}{\tau}$$

$$\frac{d\phi}{dt} = \frac{V_a \sigma c}{V}\phi N - \frac{\phi}{\tau_c}$$

Difference:
- Population inversion
- Stimulated emission term

Similarity:
- Same 2nd equation

Contents

Threshold state: N_c and R_{cp} (4L)

$$\frac{dN}{dt} = R_p - B\phi N - \frac{N}{\tau}$$
$$\frac{d\phi}{dt} = V_a B\phi N - \frac{\phi}{\tau_c}$$

Note: Small amount of photons ϕ_i exist due to spontaneous emission

In 2nd equation, let $d\phi/dt=0$: $$N_c = \frac{1}{BV_a\tau_c} = \frac{\gamma}{\sigma l}$$

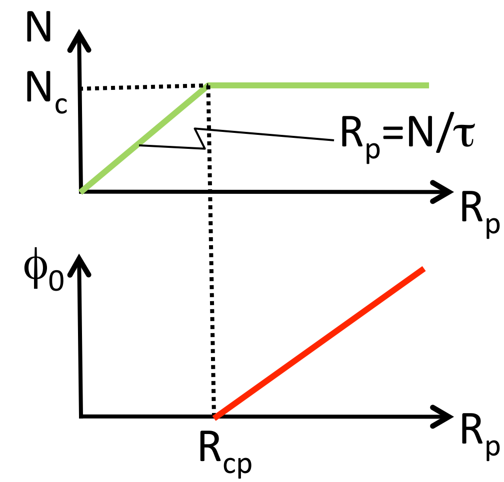
Physically: gain=loss

In 1st equation, let $dN/dt=0$, $\phi\approx 0$, and $N=N_c$: $$R_{cp} = \frac{N_c}{\tau} = \frac{\gamma}{\sigma l\tau}$$

Steady state: N_0, ϕ_0, P_{out}, η_s (4L)

$$\boxed{\begin{aligned}\frac{dN}{dt} &= R_p - B\phi N - \frac{N}{\tau}\\ \frac{d\phi}{dt} &= V_a B\phi N - \frac{\phi}{\tau_c}\end{aligned}}$$

Steady lasing state: constant $R_p > R_{cp}$



In 2nd equation, let $d\phi/dt=0$: $\boxed{N_0 = \frac{1}{BV_a\tau_c} = \frac{\gamma}{\sigma l}}$

In 1st equation, let $dN/dt=0$, and $N=N_0$:

$$\boxed{\phi_0 = V_a\tau_c\left(R_p - \frac{N_0}{\tau}\right) = V_a\tau_c\left(R_p - R_{cp}\right)}$$

$$\phi_0 = V_a N_0 \frac{\tau_c}{\tau}\left(\frac{R_p}{R_{cp}} - 1\right) = V_a N_0 \frac{\tau_c}{\tau}\left(\frac{P_p}{P_{th}} - 1\right)$$

The output power $\boxed{P_{out} = \phi_0 \frac{\gamma_2 c}{2L_e} h\nu = (A_b I_s)\frac{\gamma_2}{2}\left(\frac{P_p}{P_{th}} - 1\right)}$

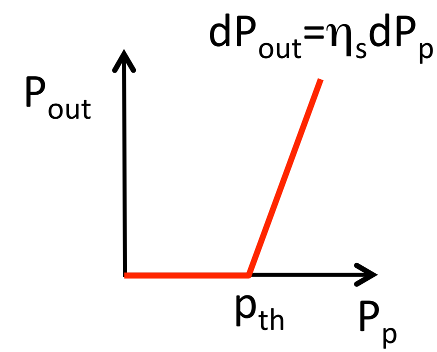
N_0 and τ_c are known

I_s: saturation intensity $I_s = \frac{h\nu}{\sigma\tau}$

A_b: laser beam cross-section area $A_b = \frac{V_a}{l}$

Steady state: N_0, ϕ_0, P_{out}, η_s (4L)

Slope efficiency $$\eta_s = \frac{dP_{out}}{dP_p} = (A_b I_s)\,\frac{\gamma_2}{2}\,\frac{1}{P_{th}}$$



Special case: lamp and diode (transverse) pumping
(active medium is uniformly pumped)

Since $$\boxed{P_{th} = \frac{\gamma}{\eta_p}\,\frac{h\nu_{mp}}{\tau}\,\frac{A}{\sigma}}$$

$$R_{cp} = \frac{\gamma}{\sigma l \tau}$$

$$R_{cp} = \eta_p \frac{P_{th}}{Alh\nu_{mp}}$$

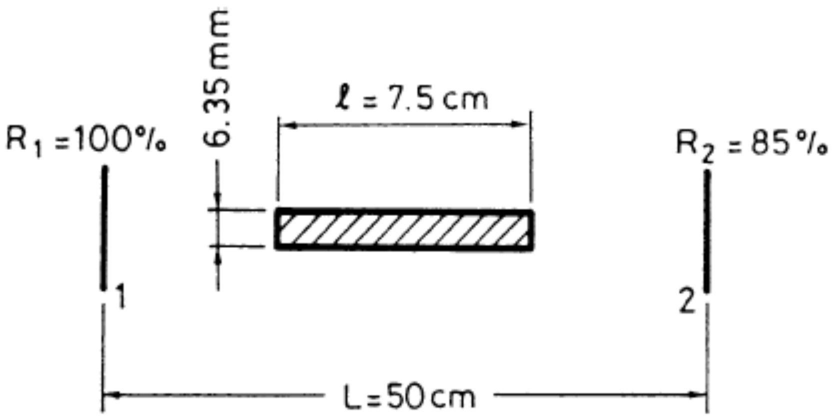
$$\begin{aligned}\eta_s &= \eta_p \cdot \frac{\gamma_2}{2\gamma} \cdot \frac{h\nu}{h\nu_{mp}} \cdot \frac{A_b}{A} \\ &= \eta_p \cdot \eta_c \cdot \eta_q \cdot \eta_t\end{aligned}$$

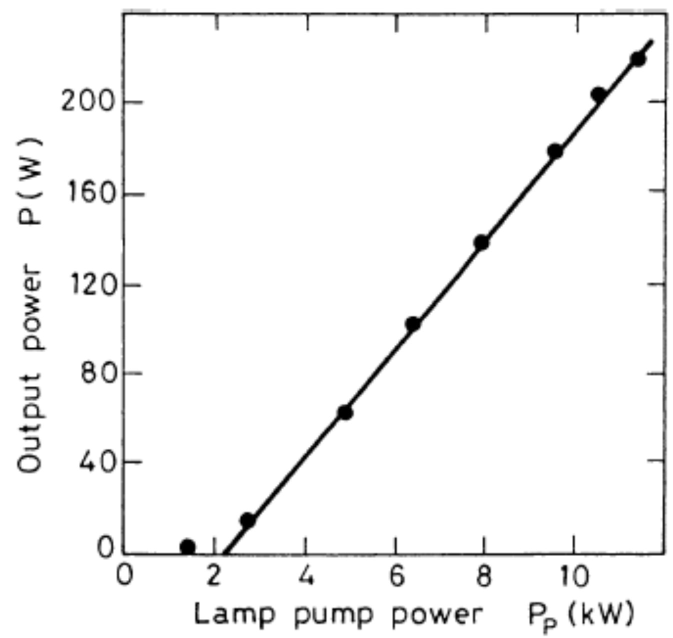
Transverse efficiency
Quantum efficiency
Output coupling efficiency
Pump efficiency

A: cross-sectional area of the pumped region of the active medium

Nd:YAG example

1% atomic doping, lamp-pumped





- Multi-mode → Space-independent model approximately valid
- P_{th}=2.2kW
- η_s=2.4%
- $N_c \approx 5.7\times10^{16}$ions/cm^3; $N_{tot}=4.1\times10^{20}$ions/cm^3; PI fraction: 0.04%

Q: how to calculate N_c, from the figure and other parameters?

Threshold and steady states (q3L)

$$N_c = \frac{V}{V_a \sigma_e c \tau_c} = \frac{\gamma}{\sigma_e l} \quad \leftarrow \left[\frac{d\phi}{dt} = 0\right]$$

$$R_{cp} = \frac{f N_t + N_c}{(1+f)\tau} \quad \leftarrow \left[\frac{dN}{dt} = 0, \phi = 0, N = N_c\right]$$

$$P_{th} = \frac{h\nu_p}{\eta_p \tau} \frac{(f N_t + N_c) A l}{1+f} \quad \leftarrow \left[\frac{dN}{dt} = \frac{d\phi}{dt} = 0\right]$$

$$= \frac{\gamma(1+B)}{\eta_p} \frac{h\nu_p}{\tau} \frac{A}{\eta_e + \eta_a}$$

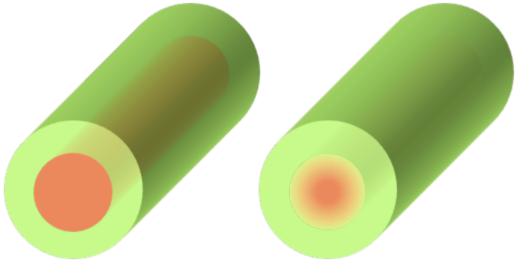
$$P_{out} = \frac{A_b(1+B)}{\eta_e + \eta_a} \frac{h\nu}{\tau} \frac{\gamma_2}{2} \left(\frac{P_p}{P_{th}} - 1\right)$$

$$\eta_s = \frac{dP_{out}}{dP_p} = \eta_p \cdot \frac{\gamma_2}{2\gamma} \cdot \frac{h\nu}{h\nu_p} \cdot \frac{A_b}{A}$$

Spatial-dependent case

Pump and laser mode densities are **not** uniform

Consequence: Rp, |u|², N are no longer uniform.



Threshold conditions:

$$\langle N \rangle_c = \frac{\gamma}{\sigma l}$$

$$\langle R_p \rangle_c = \frac{\langle N \rangle_c}{\tau} = \frac{\gamma}{\sigma l \tau}$$

$$\langle N \rangle_0 = \langle N \rangle_c = \frac{\gamma}{\sigma l}$$

- P_{th}: depends on w_0, and w_p (if longitudinal-diode pumping) or a (if transverse pumping)
- P_{out}: depends on w_0, and w_p or a
- η_s: (especially η_t) depends on w_0, and w_p or a

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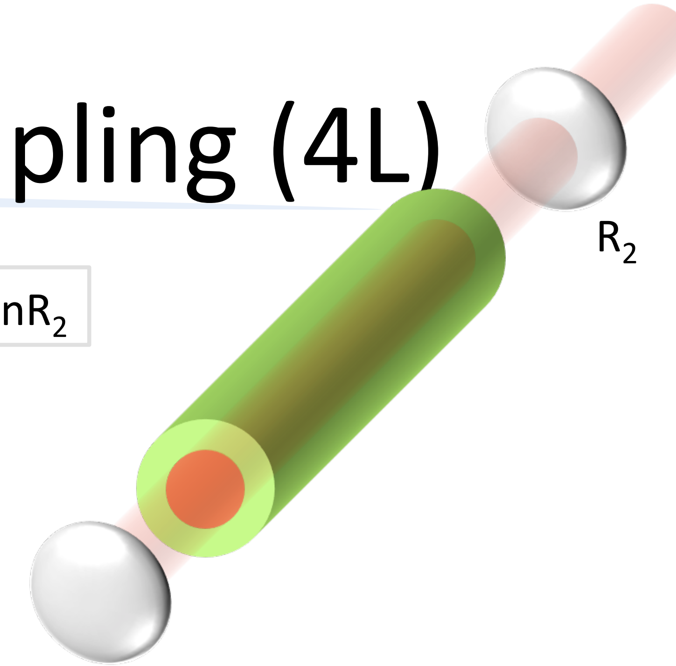
R_2 for optimum coupling (4L)



$$\boxed{P_{out} = \phi_0 \frac{\gamma_2 c}{2L_e} h\nu = (A_b I_s) \frac{\gamma_2}{2}\left(\frac{P_p}{P_{th}} - 1\right)}$$

$\gamma_2 = -\ln R_2$

Condition: $\dfrac{dP_{out}}{d\gamma_2} = 0$

or $\dfrac{dP_{out}}{dS} = 0$ where $S = \frac{\gamma_2}{2\gamma_i + \gamma_1}$

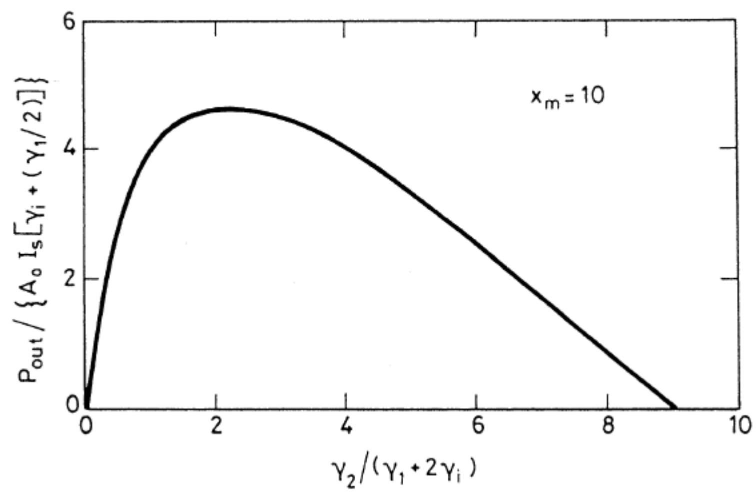
Optimum state:

$$P_{op} = \left[A_b I_s \left(\gamma_i + \frac{\gamma_1}{2}\right)\right] \left(\sqrt{x_m} - 1\right)^2$$

$$S_{op} = \sqrt{x_m} - 1$$

where $x_m = \dfrac{P_p}{P_{mth}}$

P_{mth} is minimum threshold pump power (γ_2=0)



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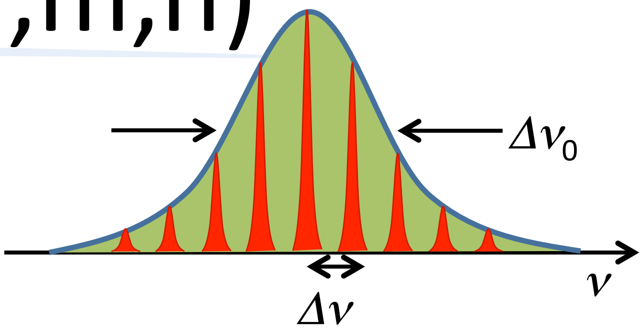
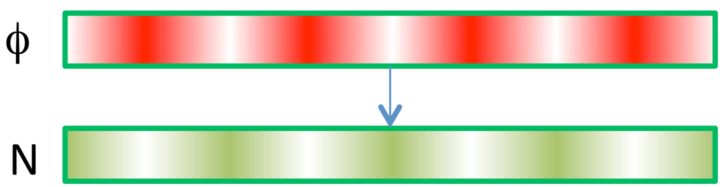
Multimodeness (l,m,n)



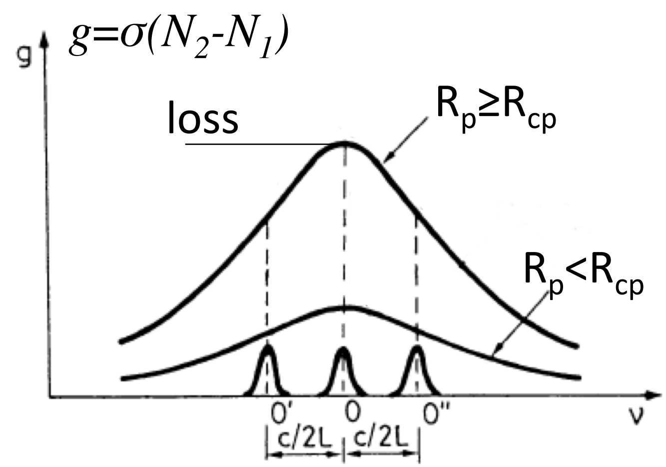
Fact: $\Delta\nu << \Delta\nu_0$

Reason:
L=1m → $\Delta\nu$=150MHz
while $\Delta\nu_0$=1~300GHz

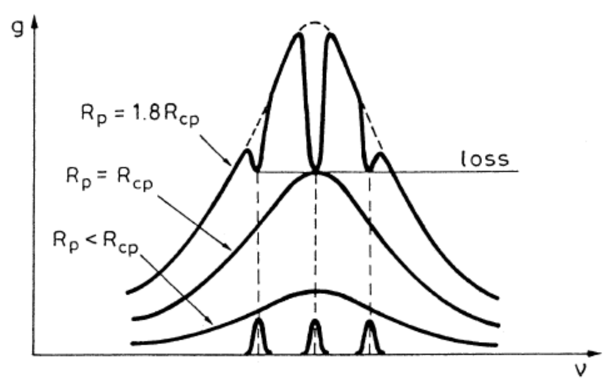
- **Homogeneously-broadened gain line**



Spatial hole burning



- **Inhomogeneously-broadened gain line**



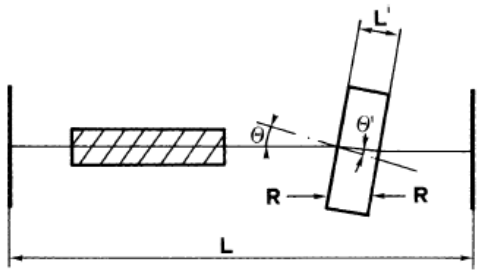
Spectral hole burning

Comments:

- Spatial hole burning does not apply for inhomogeneous-line case
- Homogeneous-line case: a few modes around the gain center survive

Single-mode selection

Mode discrimination



- **Single transverse-mode selection** (l,m)
  - Aperture (Fresnel no. $a^2/(c\lambda) < 2$)
  - Unstable resonators (if active rod has large Φ)
- **Single longitudinal-mode selection** (n)
  - Shorter cavity? ($\Delta\nu \geq \frac{\Delta\nu_0}{2} \rightarrow \boxed{L \leq \frac{c}{\Delta\nu_0}}$)
  - Fabry-Pérot etalon
  - Ring resonators

$$\frac{\Delta\nu_{\mathrm{fsr}}}{F} \leq 2\Delta\nu$$

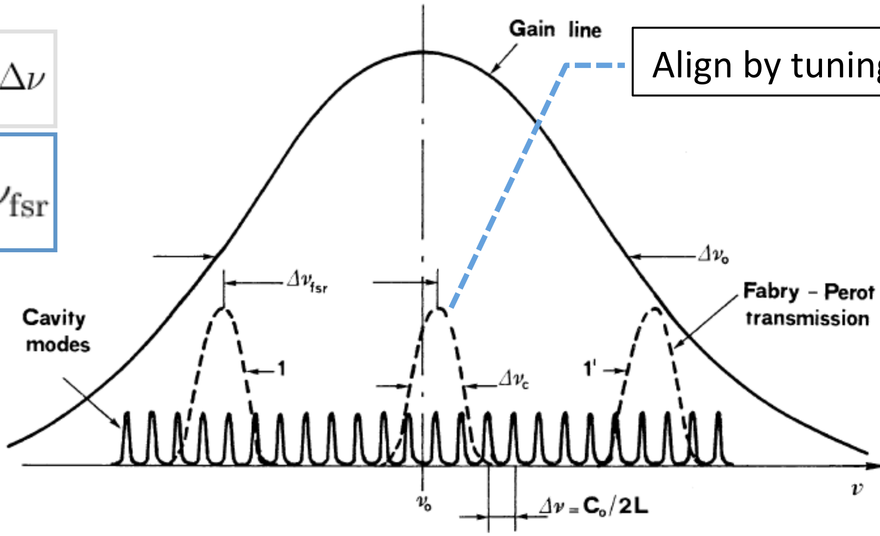
Condition: $\boxed{\frac{\Delta\nu_c}{2} \leq \Delta\nu, \quad \frac{\Delta\nu_0}{2} \leq \Delta\nu_{\mathrm{fsr}}}$

$$\frac{\Delta\nu_0}{2} \leq \Delta\nu_{\mathrm{fsr}} \leq 2F\Delta\nu$$

Necessary condition:

$$\frac{\Delta\nu_0}{2} \leq 2F\Delta\nu = 2F\frac{c}{2L}$$

$$\boxed{L \leq \frac{2Fc}{\Delta\nu_0}}$$



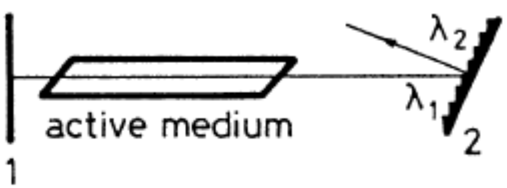
- L is relaxed by a factor of 2F
- Multiple FPs can be used

Laser tuning

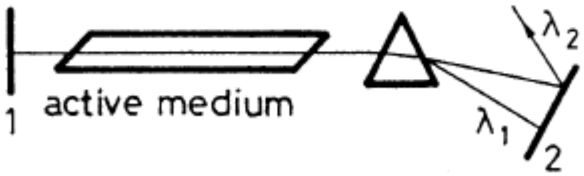
Mode discrimination

Motivation:

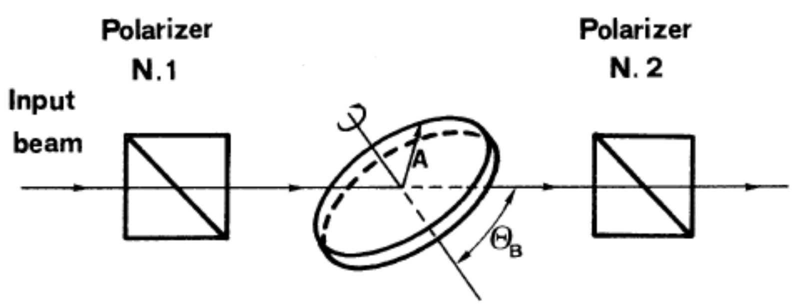
- To harness wide gain linewidth $\Delta\nu_0$ (dye or vibronic solid-state lasers)
- To lase at one of the many transition lines



- MIR lasers
- Tuning: grating rotation

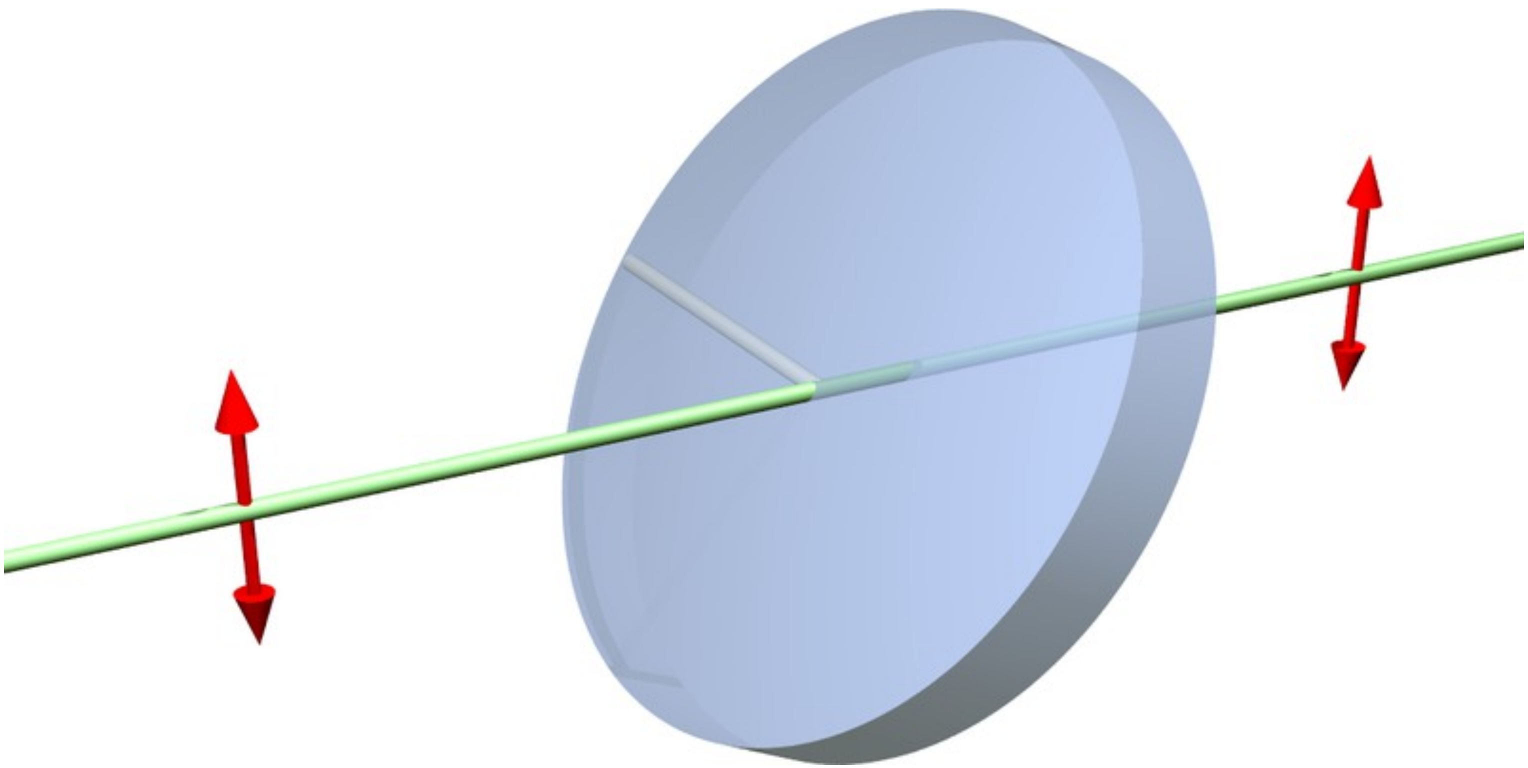


- VIS-NIR lasers
- Tuning: prism rotation



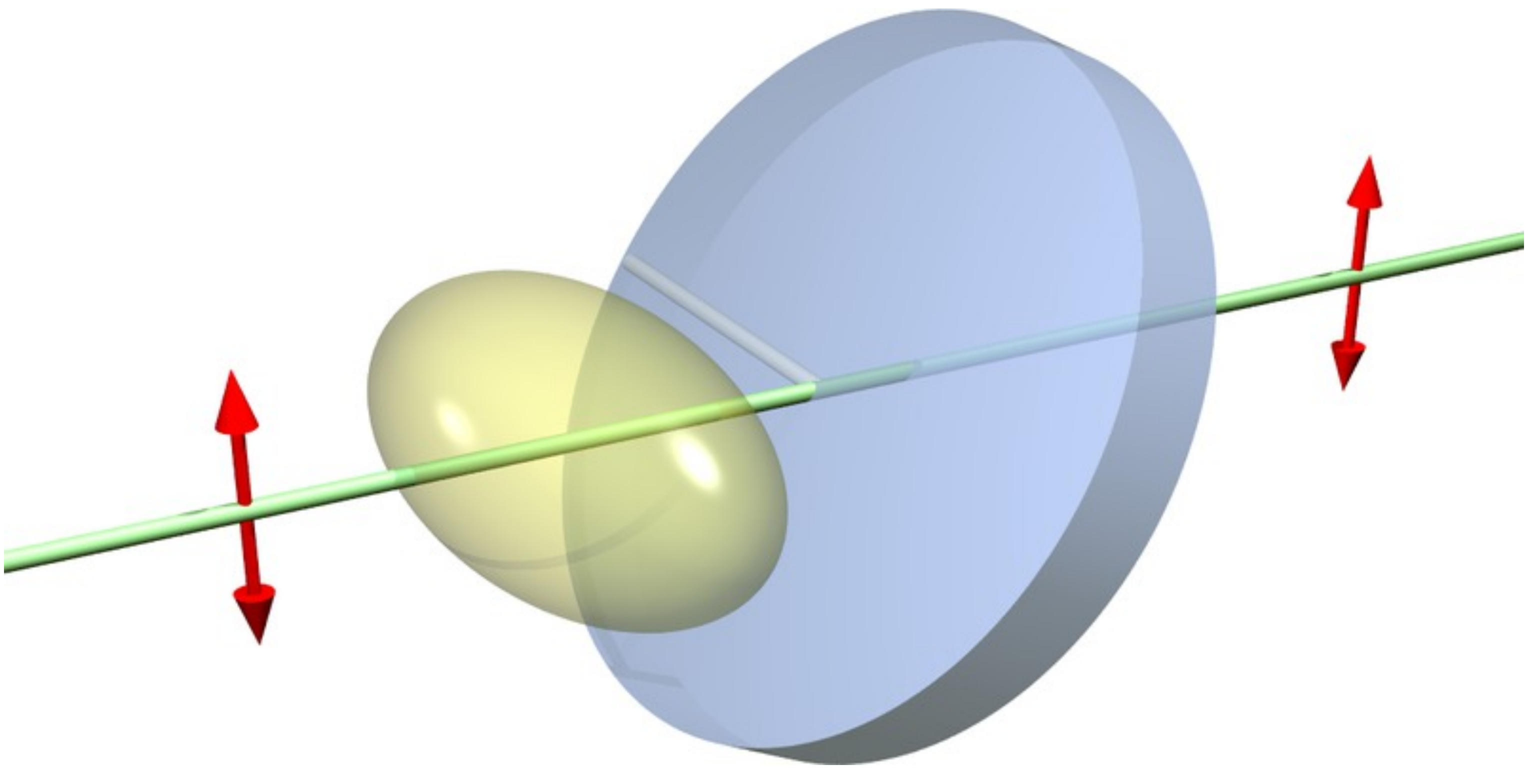
Laser tuning

Φ=0°

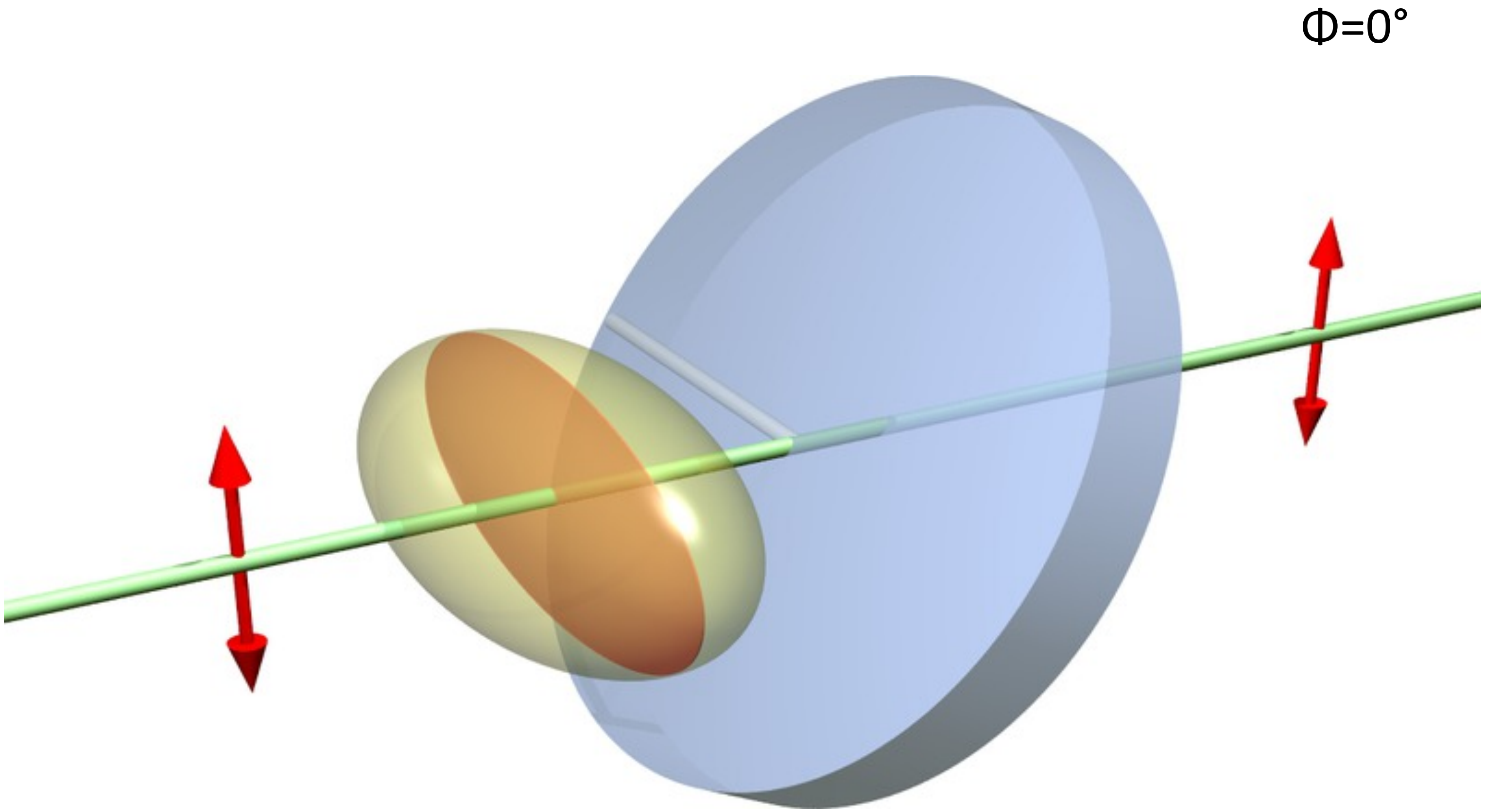


Laser tuning

$\Phi = 0°$

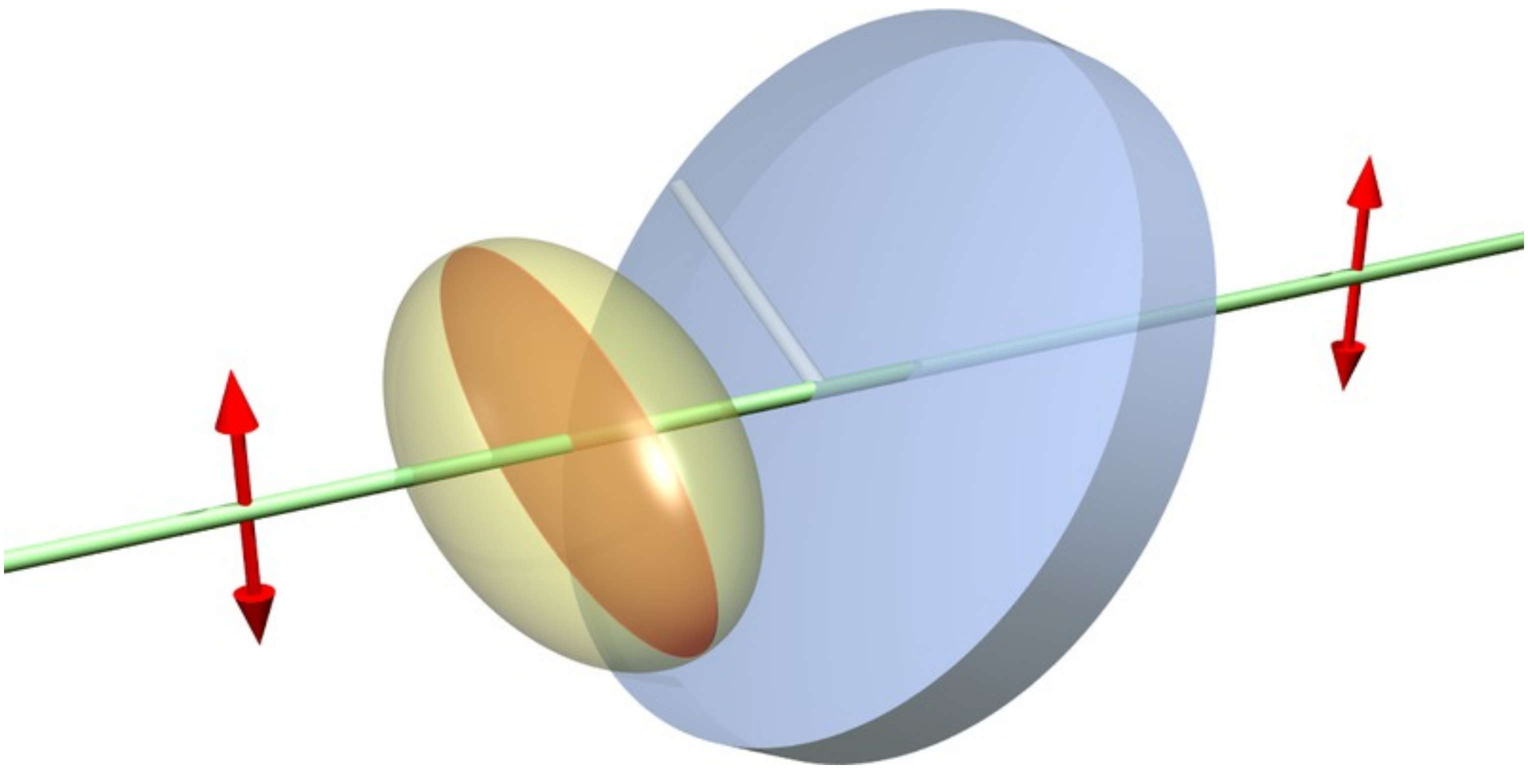


Laser tuning



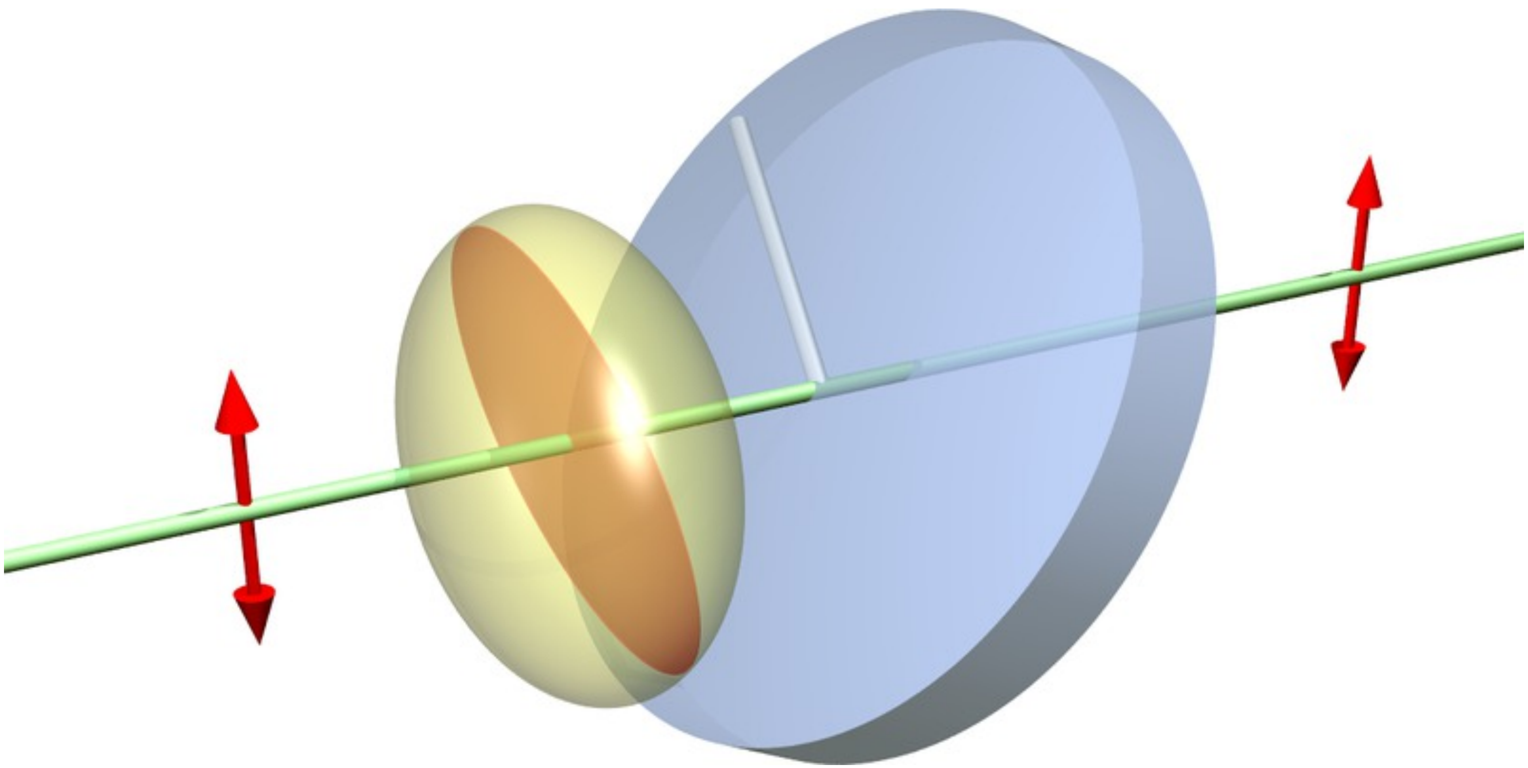
Laser tuning

Φ=15°



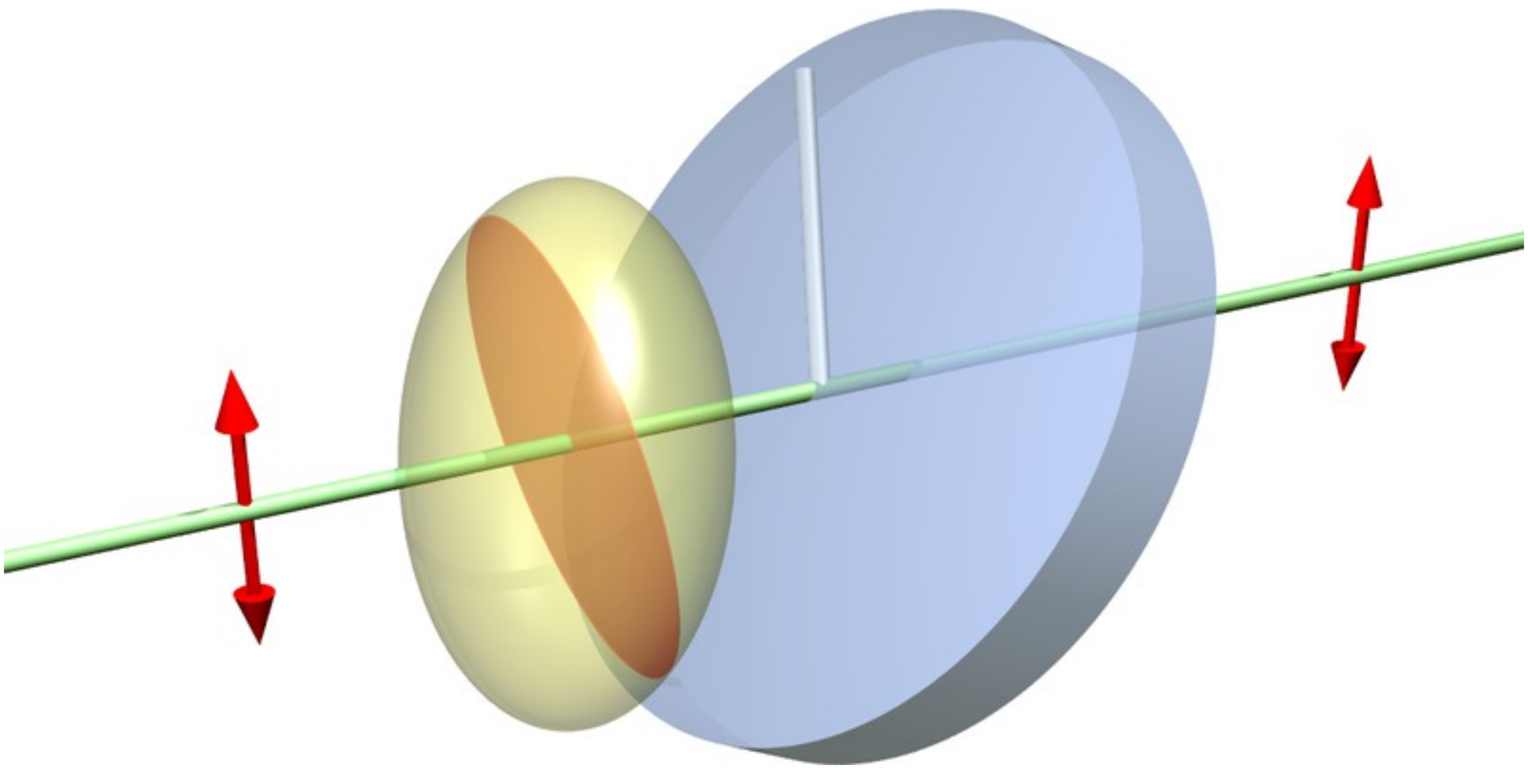
Laser tuning

Φ=30°



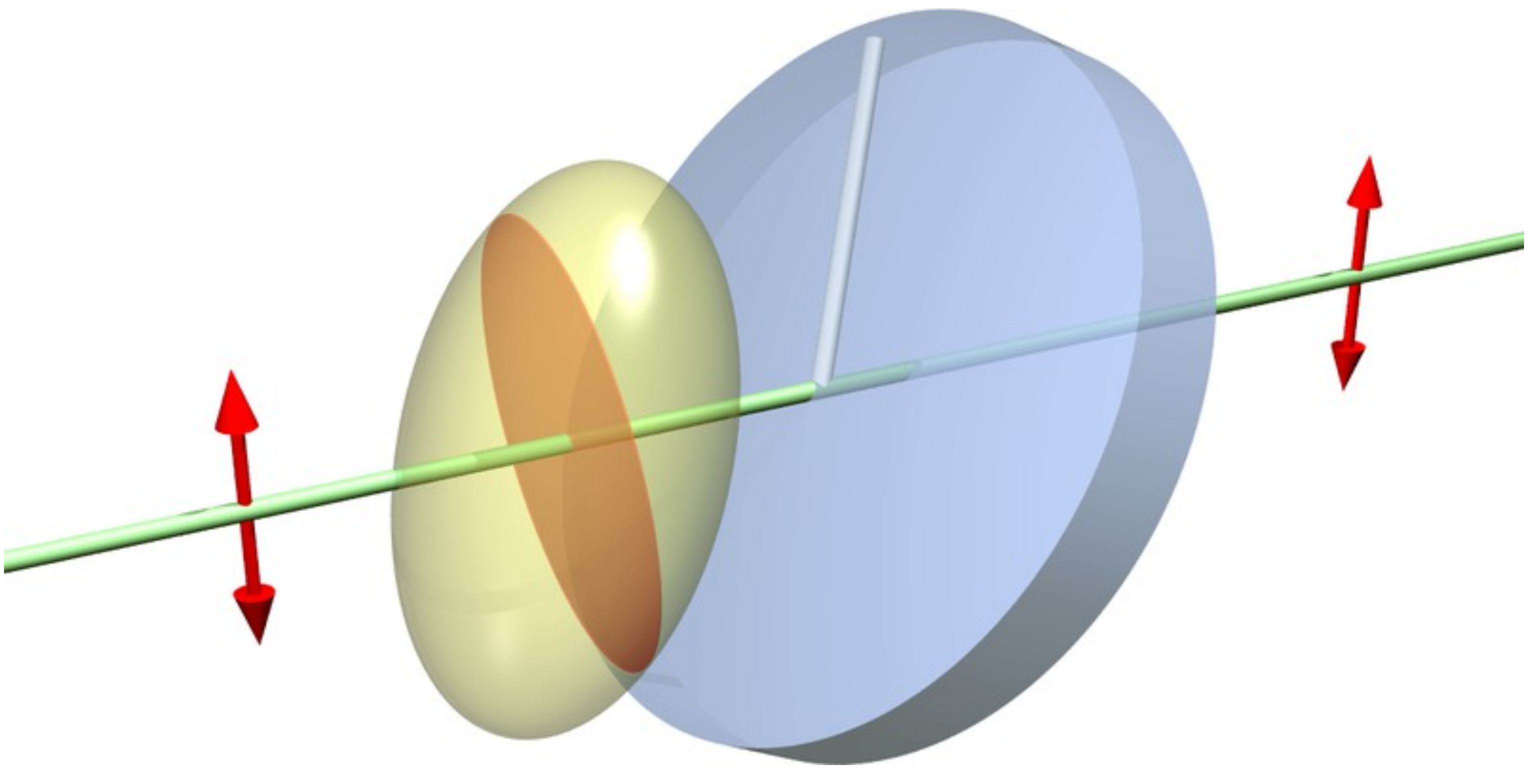
Laser tuning

Φ=45°



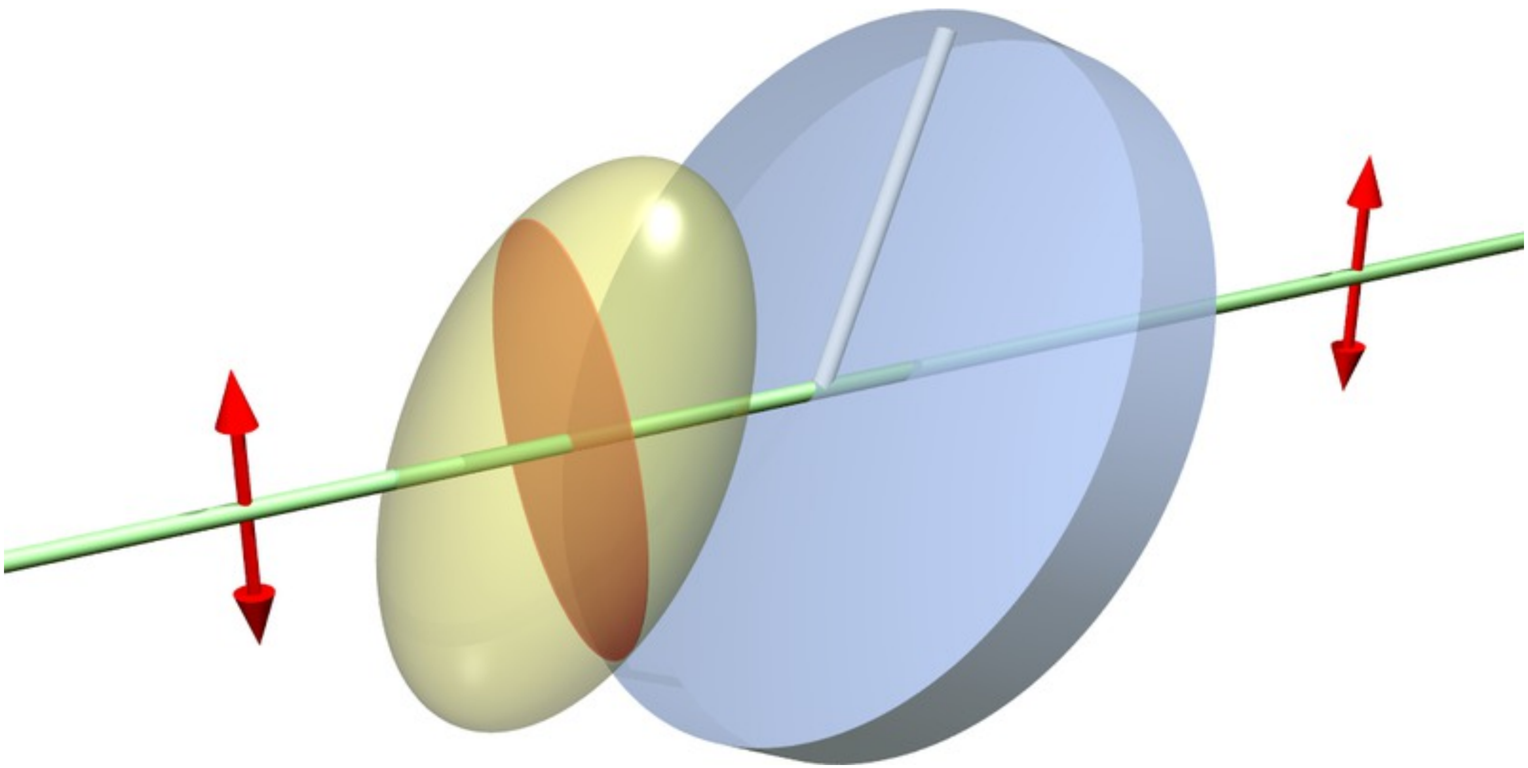
Laser tuning

$\Phi=60°$



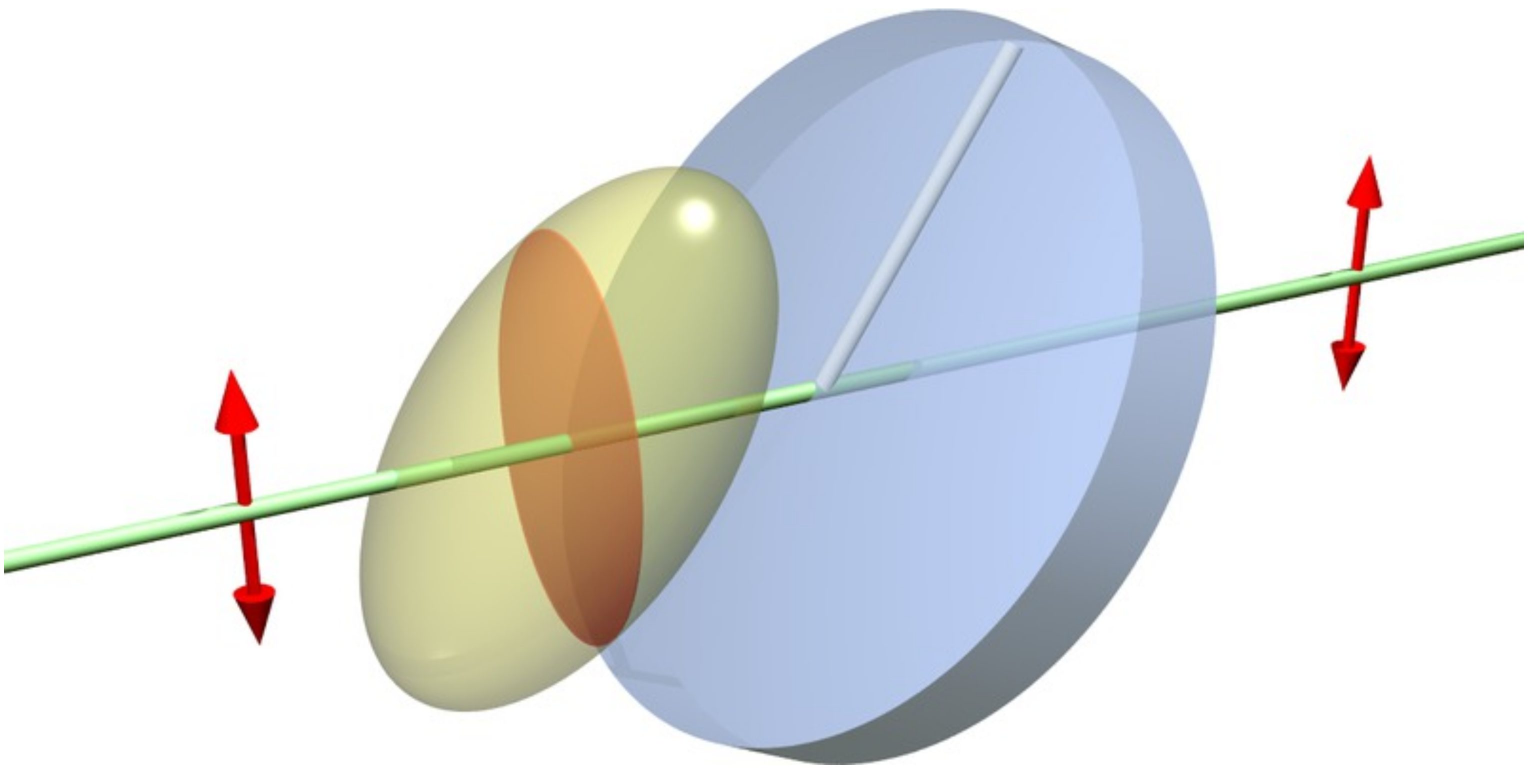
Laser tuning

Φ=75°



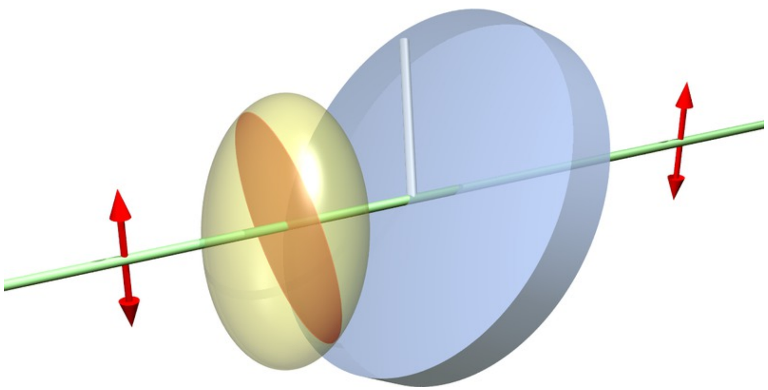
Laser tuning

Φ=90°



Laser tuning

Φ=45°



- After 1st polarizer Incidence is first split into e- and o-rays

$$E_e = E_i \frac{\sqrt{2}}{2} \qquad E_o = E_i \frac{\sqrt{2}}{2}$$

- After prop. through plate

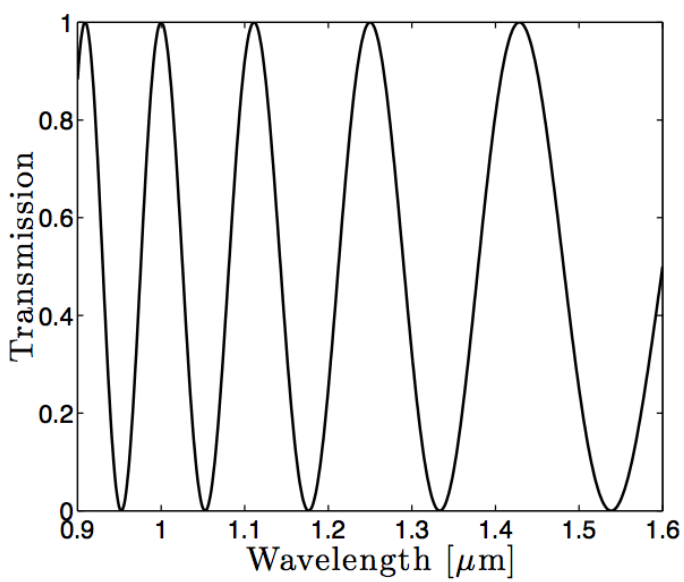
$$E_e' = E_i \frac{\sqrt{2}}{2} \cos \Delta\phi \qquad E_o' = E_i \frac{\sqrt{2}}{2}$$

- After 2nd polarizer

$$E_{out} = \frac{1}{2} E_i \cos \Delta\phi + \frac{1}{2} E_i$$
$$= E_i \cos^2 \left(\frac{\Delta\phi}{2} \right)$$

$$\boxed{T = \cos^2 \left(\frac{\Delta\phi}{2} \right) = \cos^2 \left[\frac{\pi}{\lambda} (n_e - n_o) L_e \right]}$$

n_e=1.46
n_o=1.45
L_e=1mm



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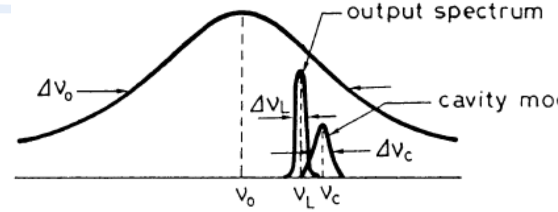
Others



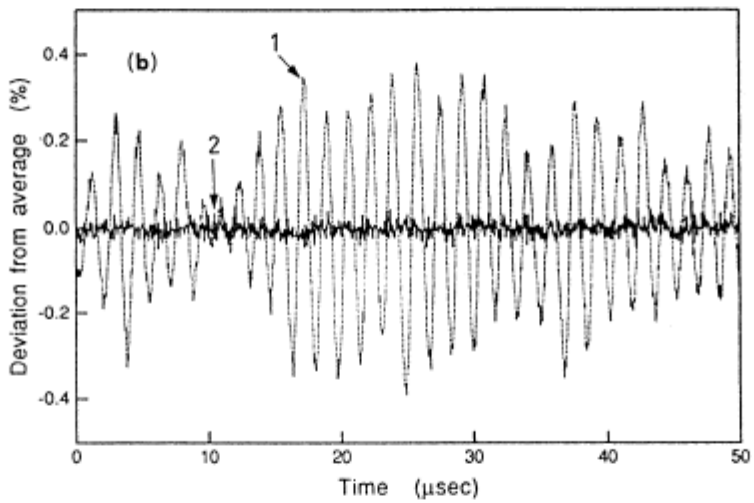
$$\nu_L = \frac{\frac{\nu_0}{\Delta\nu_0} + \frac{\nu_c}{\Delta\nu_c}}{\frac{1}{\Delta\nu_0} + \frac{1}{\Delta\nu_c}}$$

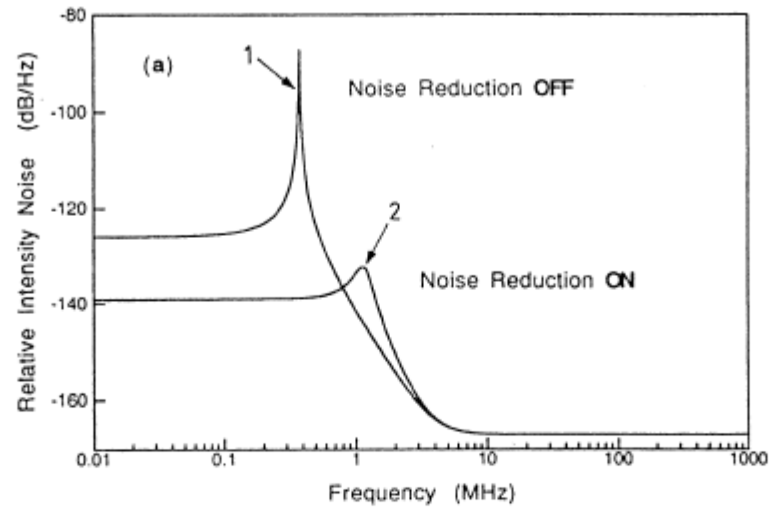
- **Frequency pulling**
  - Formula is exact for homogeneous line
  - $\nu_L \approx \nu_0/1000 + \nu_c$ [relatively small]

- **Frequency fluctuation** [cavity length: $L_e = n(L-1)+n_a l$]
  - Long-term (>1s): T, ambient pressure
  - Short-term (<1s): mirror vibration, n or n_a change, acoustic wave
  - Stabilization: passive (isolation) or active (feedback system)

- **Intensity noise**
  - Gas: Pp, discharge, cavity
  - Dye: jet density, bubbles
  - Solid-state: Pp, cavity
  - Semiconductor: I_{bias}, E-H recombination noise
  - Reduction: feedback system

Intensity noise





Contents

Content	Time
1. Rate equations 1. Four-level; 2. Quasi-three-level	25’
2. Threshold and steady states 1. Four-level; 2. Quasi-three-level	15’
3. Optimum output coupling	5’
4. Tuning and single-mode selection	30’
5. Others Frequency-pulling, fluctuations	5’
Total:	80’