Lecture 7
Pumping & Population Inversion*

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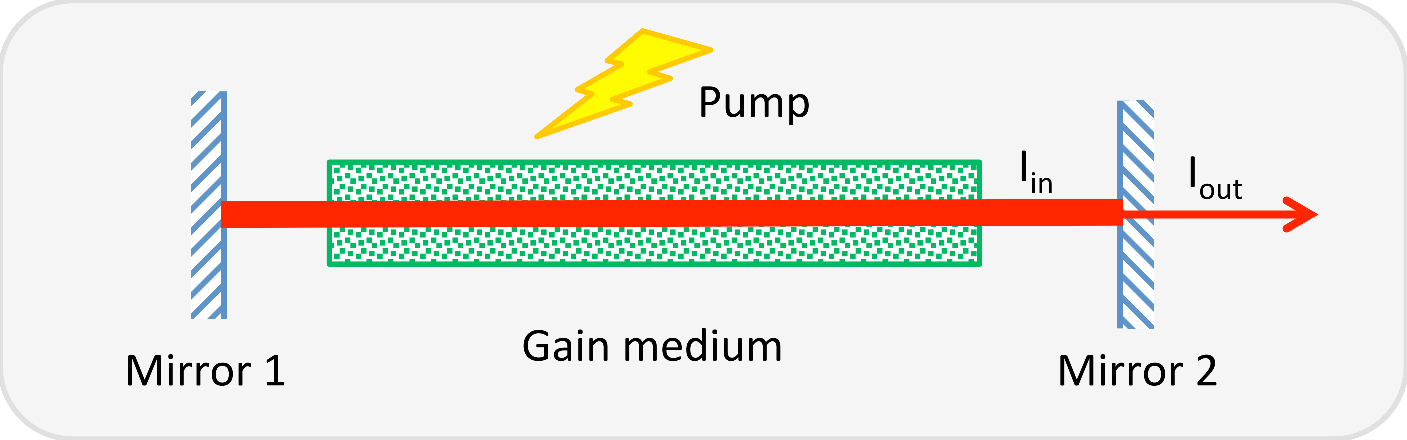
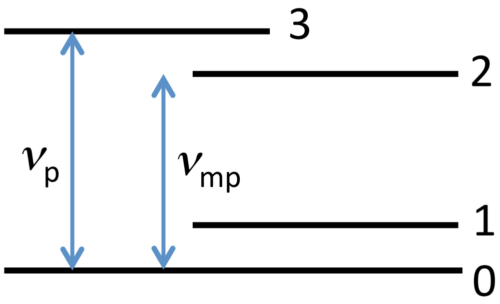
[*] Some figures and texts belong to: O. Svelto, *Principles of Lasers*, 5th Ed., Springer.

Reading

- *Principles of Lasers* (5th Ed.): Chapter 6.
- Skip: Quantum mechanical treatment in 6.4.1.1
- Squeeze: 6.3.3-6.3.5, 6.4.1-6.4.4.

Laser





- Pump efficiency: $\eta_p = P_m / P_p$
- Pump rate: R_p, or dN_2/dt
- Threshold pump power: P_{th}

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1. Pumping varieties	10′
2. Optical pumping (lamp) 2.1. Schemes; 2.2 η_p and R_p	20′
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4. Electrical pumping	25′
Total:	**80′**

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Pumping varieties

- **Optical pumping** (CW/pulsed)
  - **Lamp**: for solid-state/liquid lasers [broad absorption bands]
  - **Laser**: for solid-state/liquid/gas lasers [broad/narrow absorption bands/lines]

- **Electrical pumping** (CW/RF/pulsed; for gas/semiconductor lasers)

- **X-ray pumping**

- **E-beam pumping**

- **Chemical pumping**

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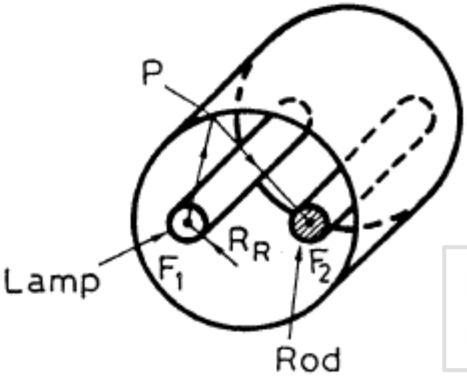
Pumping: Lamp

Lamp types

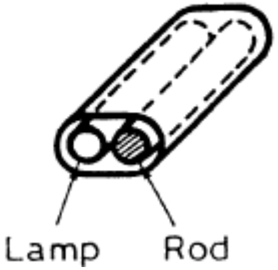
- *For pulsed lasers:*
  Medium-to-high pressure (500~1500 Torr) Xe or Kr flashlamps
- *For CW lasers:*
  High-pressure (1-8atm) Kr lamps

- **Elliptical cylinder**
  - Rod and lamp at foci
  - Specular reflection



Rod radius: mm~cm
Rod length: cm~<1m

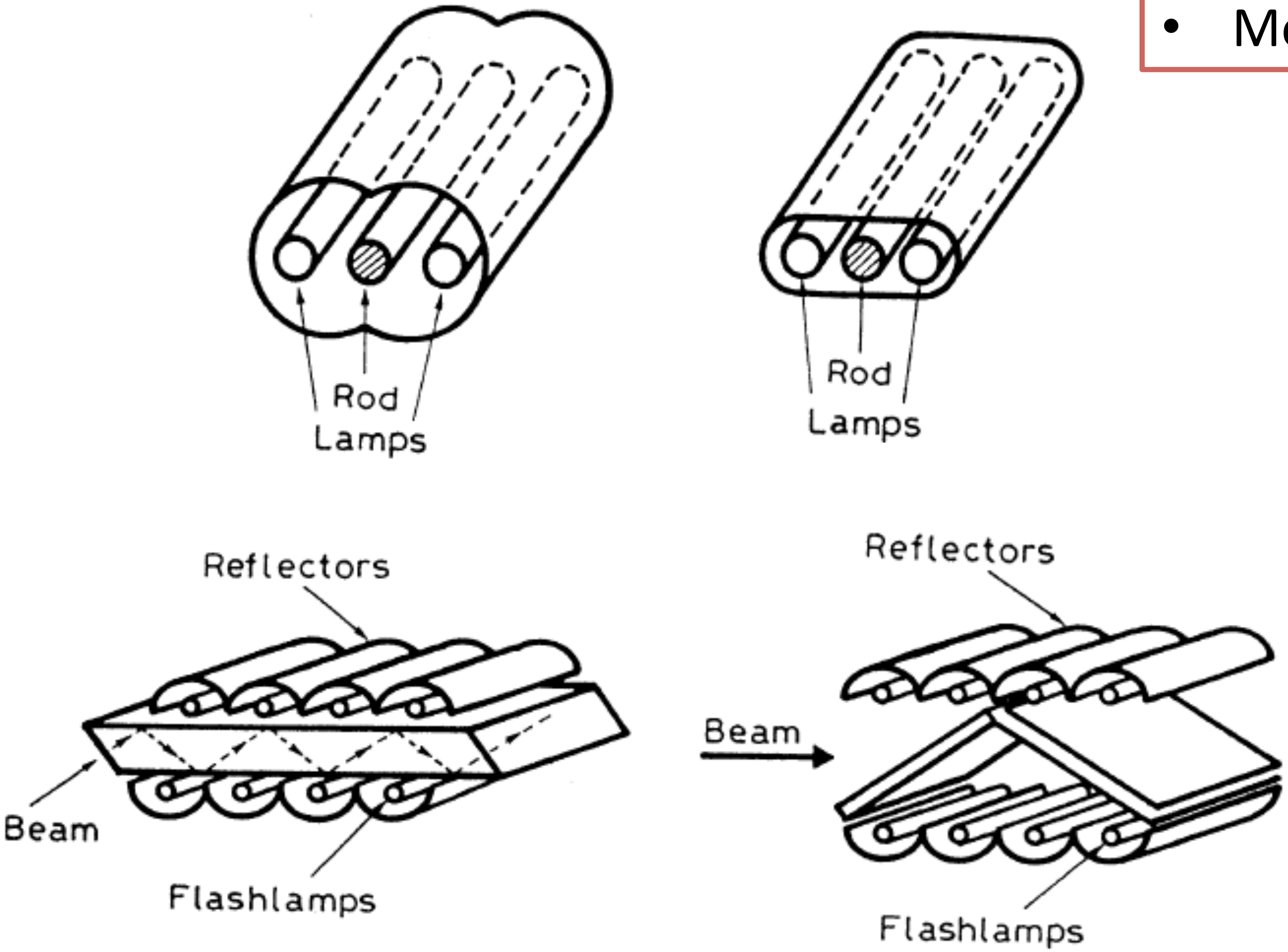
- **Close-coupling**
  - Close-pack
  - Diffusive reflection
  - ✓ More uniform



1 atm = 760 Torr

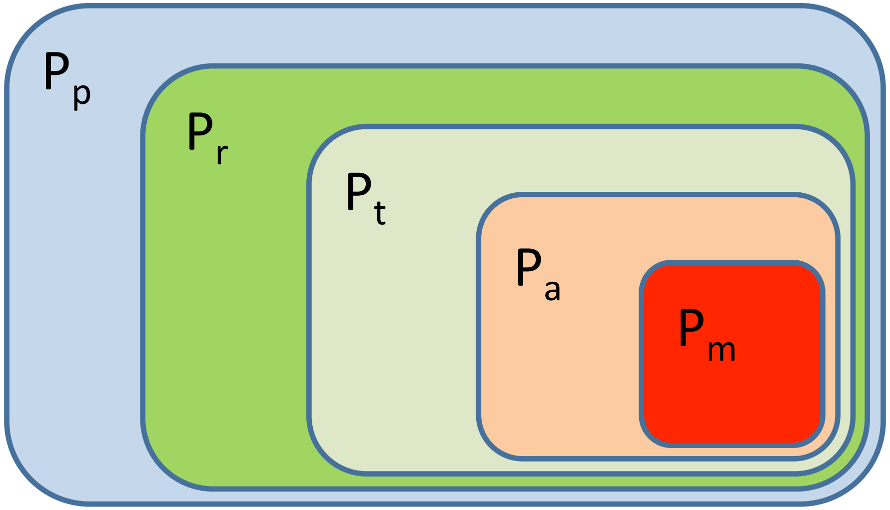
Pumping: Lamps

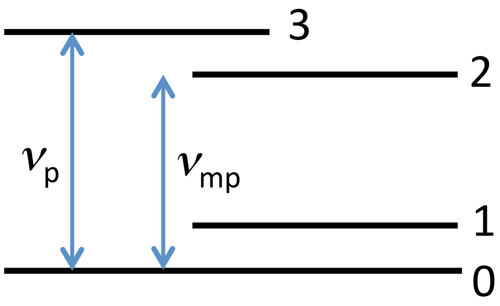
- $P_p \uparrow$ $\eta_p \downarrow$
- More uniform



Pump efficiency and pump rate

Assumption: uniform R_p





$$P_m = \left(\frac{dN_2}{dt}\right)_p V h\nu_{mp} = R_p V h\nu_{mp}$$

$$P_m = P_p\eta_p$$

$$\boxed{R_p = \eta_p \frac{P_p}{Alh\nu_{mp}}}$$

$$\eta_p = \frac{P_r}{P_p}\frac{P_t}{P_r}\frac{P_a}{P_t}\frac{P_m}{P_a} = \eta_r\eta_t\eta_a\eta_{pq}$$

- Radiative efficiency
- Transfer efficiency
- Absorption efficiency
- Power quantum efficiency

Pump efficiency, η_p

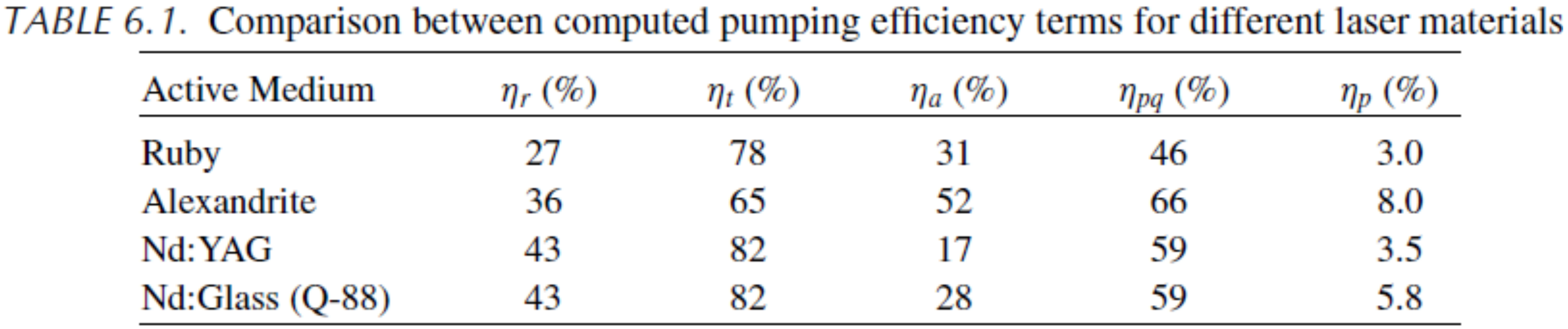


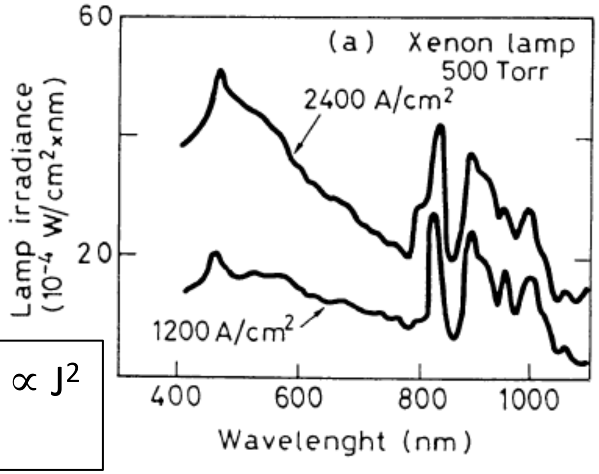
TABLE 6.1. Comparison between computed pumping efficiency terms for different laser materials

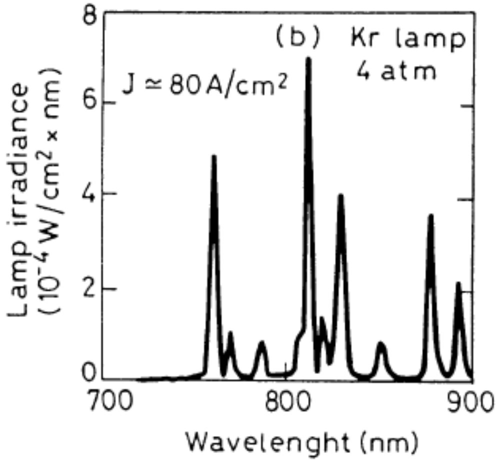
Active Medium	η_r (%)	η_t (%)	η_a (%)	η_{pq} (%)	η_p (%)
Ruby	27	78	31	46	3.0
Alexandrite	36	65	52	66	8.0
Nd:YAG	43	82	17	59	3.5
Nd:Glass (Q-88)	43	82	28	59	5.8

Configuration: Elliptical cylinder

Absorption efficiency, η_a

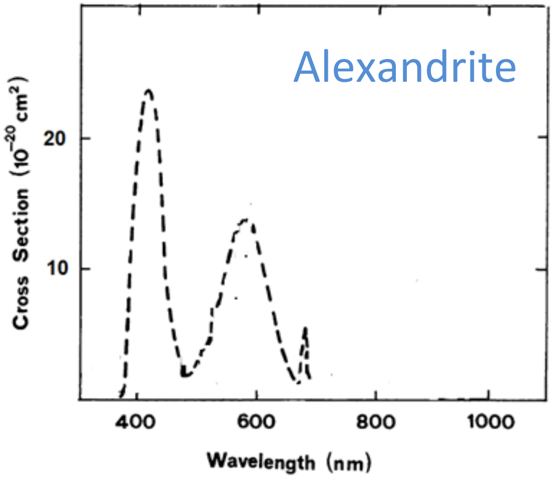
Emission

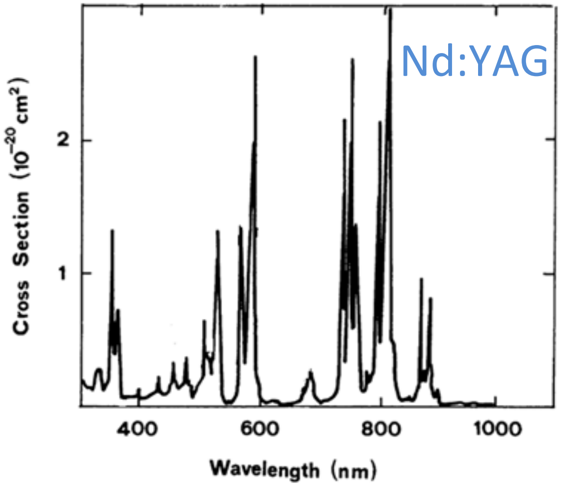




- Continuum $\propto J^2$
- Lines $\propto J$

Absorption





- Nd:YAG: Nd^{3+} in $Y_3Al_5O_{12}$ crystal (host-independent)
- Alexandrite: Cr^{3+} in $BeAl_2O_4$ crystal (host-dependent)

Contents

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1. Pumping varieties	10′
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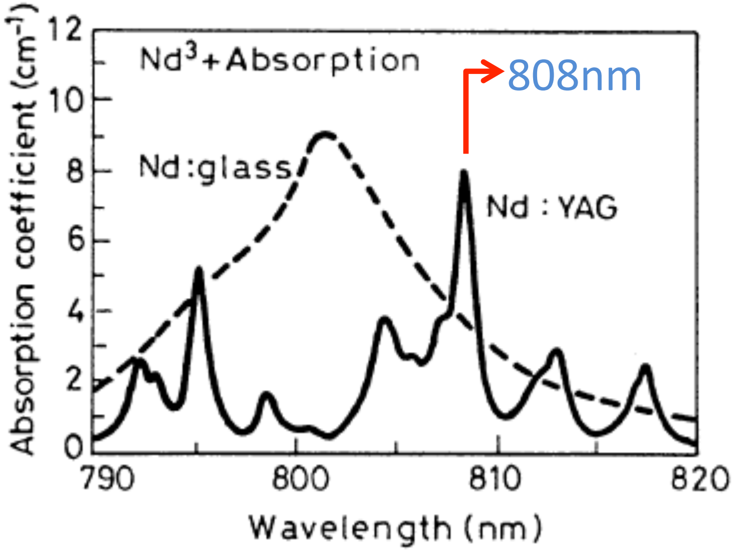
4. Electrical pumping	25′
Total:	**80′**

Laser pumping

- **Convenience:** Efficient and high-power diode lasers widely available

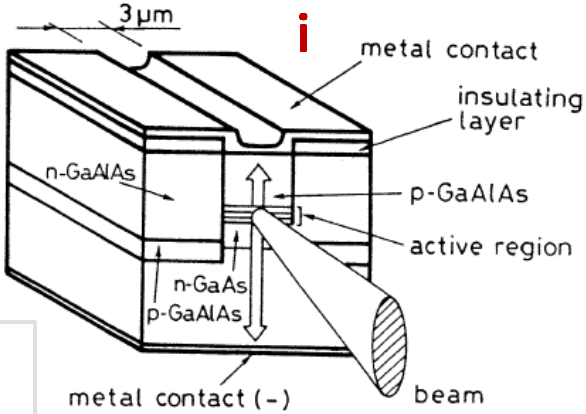
- **Key examples:**
    - **Nd:YAG** and “siblings” pumped by GaAs/AlGaAs QW-lasers (~800nm)
    - Yb:YAG, Nd:glass or Yb:Er:glass pumped by InGaAs/GaAs QW-lasers (950~980nm)
    - Alexandrite, Cr:LISAF pumped by GaInP/AlGaInP QW-lasers (640-680nm)
    - Tm:Ho:YAG laser pumped by AlGaAs QW-lasers (785nm)



808nm

Pumping diodes

i. Single-diode
Beam:2x4μm^2; P<100mW
ii. Diode-array
P≈2W
iii. Diode-bar
P=10-20W
iv. Stacked-bars
P=100W; emission bandwidth↑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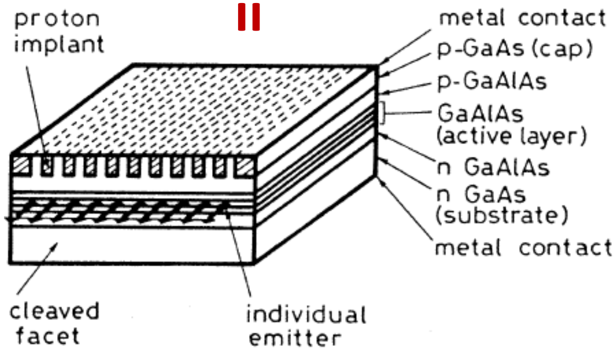
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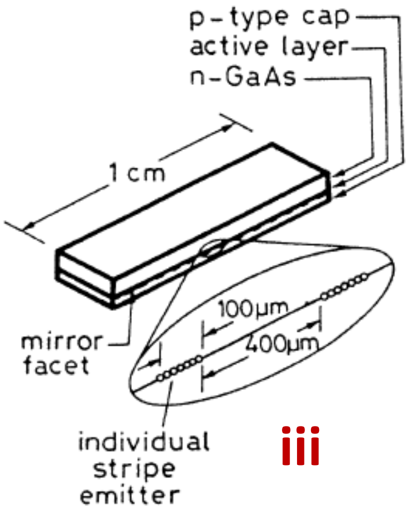
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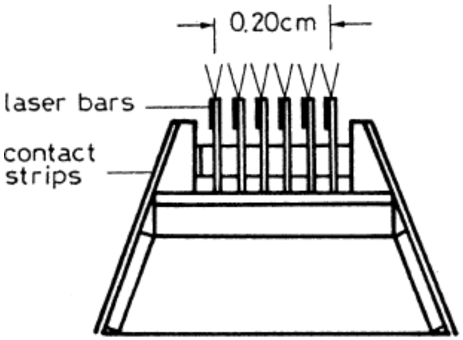
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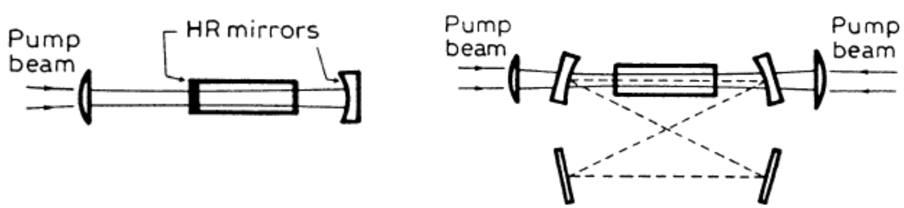
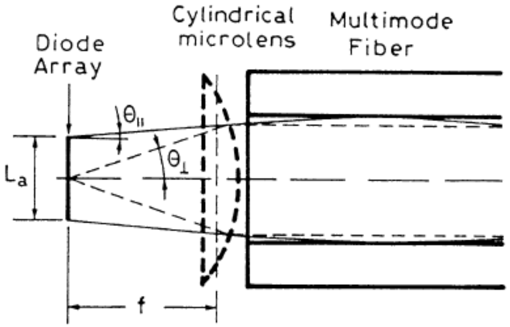
Pumping scheme

- **Longitudinal pumping**

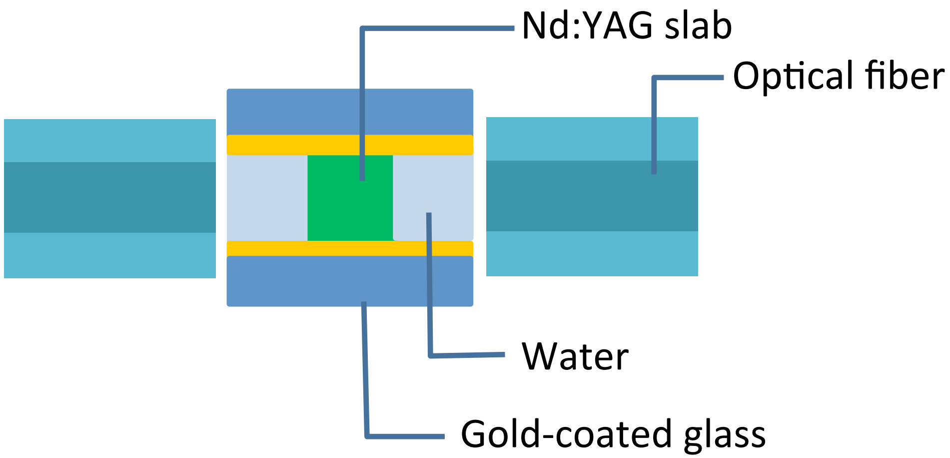


Beam shaping



Core ∅: 200µm

- **Transverse pumping**

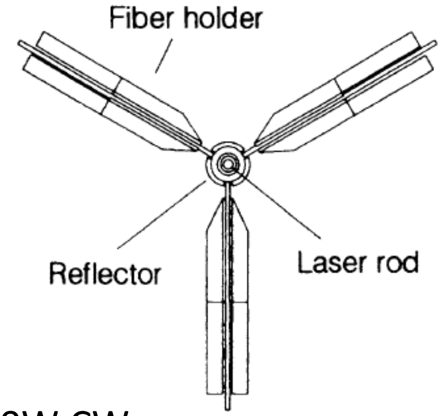


Nd:YAG slab

Optical fiber

Water

Gold-coated glass



10W CW

Pump efficiency

Nd:YAG

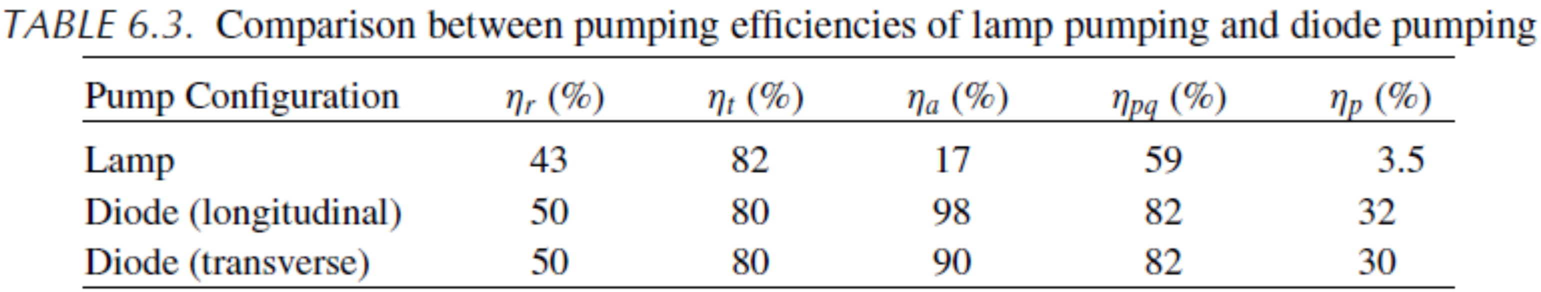


TABLE 6.3. Comparison between pumping efficiencies of lamp pumping and diode pumping

Pump Configuration	η_r (%)	η_t (%)	η_a (%)	η_{pq} (%)	η_p (%)
Lamp	43	82	17	59	3.5
Diode (longitudinal)	50	80	98	82	32
Diode (transverse)	50	80	90	82	30

- **Lamp v.s. Diode**: η_a (6×) and η_{pq} (1.5 ×)

Pump rate, $\langle R_p \rangle$

For uniform R_p

$$R_p = \eta_p \frac{P_p}{Alh\nu_{mp}}$$

Space-dependent lasing/pump beams: a matter of calculating A

1. **Diode (longitudinal)** $\langle R_P \rangle = \eta_{pd} \frac{P_p}{h\nu_{mp}} \frac{2}{\pi (w_0^2 + w_p^2) l}$

η_{pd}: η_p for diode pumping

Optimum: $w_p \approx w_0$

2. **Diode (transverse)** $\langle R_P \rangle = \eta_{pd} \frac{P_p}{h\nu_{mp}} \frac{1 - \exp\left(-\frac{2a^2}{w_0^2}\right)}{\pi a^2 l}$

Optimum *a* exists

3. **Lamp** $\langle R_P \rangle = \eta_{pl} \frac{P_p}{h\nu_{mp}} \frac{1 - \exp\left(-\frac{2a^2}{w_0^2}\right)}{\pi a^2 l}$

η_{pl}: η_p for lamp pumping

w_0: Laser beam waist
w_p: pump beam waist
a: radius of the active region

Assumption for cases 2 and 3:
- R_p=*const* if r<a (doped region)
- R_p=0 if r>a (undoped cladding)

Threshold pump power, P_{th}

Procedure:

$\langle N_2\rangle_c \rightarrow \langle R_p\rangle_c \rightarrow P_{th}$

$$\boxed{\langle N_2\rangle_c = \frac{\gamma}{\sigma_e l}}$$

$$\boxed{\langle R_p\rangle_c = \frac{\langle N_2\rangle_c}{\tau}}$$

γ: logarithmic cavity loss

σ_e: effective stimulated emission cross-section

l: active medium length

τ: upper-level lifetime

Comment: $\langle N_2\rangle_c$ differs for quasi-3-level system

1. **Diode (longitudinal)** $$P_{th} = \frac{\gamma}{\eta_{pd}} \frac{h\nu_{mp}}{\tau} \frac{\pi(w_0^2 + w_p^2)}{2\sigma_e}$$
2. **Diode (transverse)** $$P_{th} = \frac{\gamma}{\eta_{pd}} \frac{h\nu_{mp}}{\tau} \frac{\pi a^2}{\sigma_e \left[1 - \exp\left(-\frac{2a^2}{w_0^2}\right)\right]}$$
   - Can be 50% larger than case 1
3. **Lamp** $$P_{th} = \frac{\gamma}{\eta_{pl}} \frac{h\nu_{mp}}{\tau} \frac{\pi a^2}{\sigma_e \left[1 - \exp\left(-\frac{2a^2}{w_0^2}\right)\right]}$$
   - Can be 10 times larger than case 2
   - High thermal load

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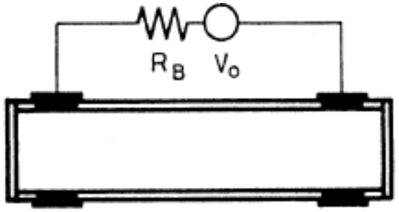
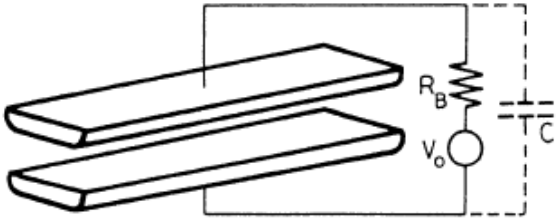
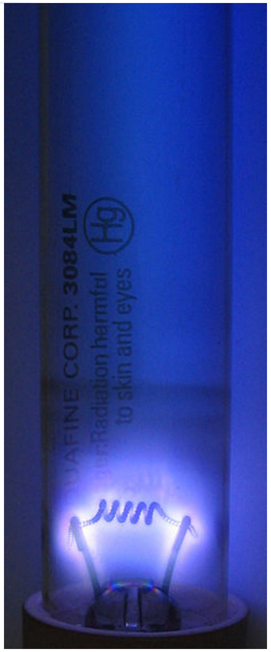
Electrical pumping

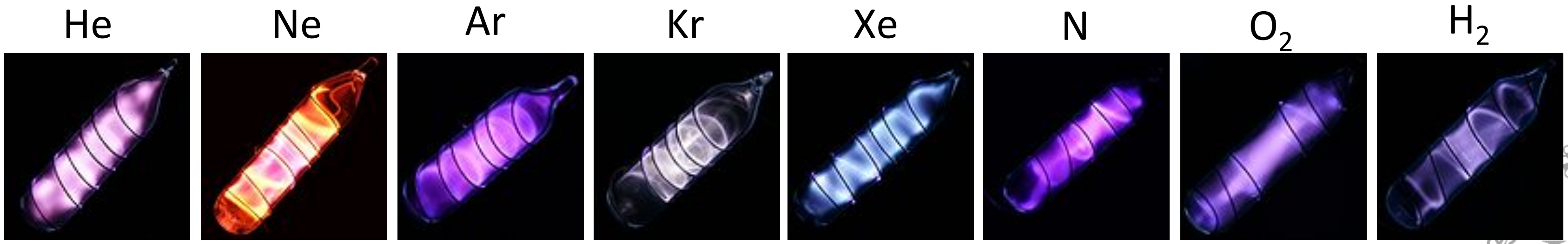
- **Applicability**: Gas or semiconductor lasers
- **Principle**: Gas discharge
- **Configuration**:
  - *Longitudinal*: Uniform pumping (glow discharge)
  - *Transverse*: small voltage (arc discharge @ edges)







Low-pressure mercury vapor discharge [Wikipedia]



Source: Wikipedia

Excitation mechanisms

- **Collision of the first kind:** $\boxed{e + \mathrm{X} \rightarrow \mathrm{X}^* + e}$
  - Or: **electron-impact excitation**
  - For single-species gas
  - More common
  - Non-resonant
  - Prelude for 2nd-kind collision

Excited state:
- Vibrational
- Rotational
- Electronic

- **Collision of the second kind:** $\boxed{\mathrm{A}^* + \mathrm{B} \rightarrow \mathrm{A} + \mathrm{B}^* - \Delta E}$
  - Resonant
  - $\Delta E < kT$ such that probability is high

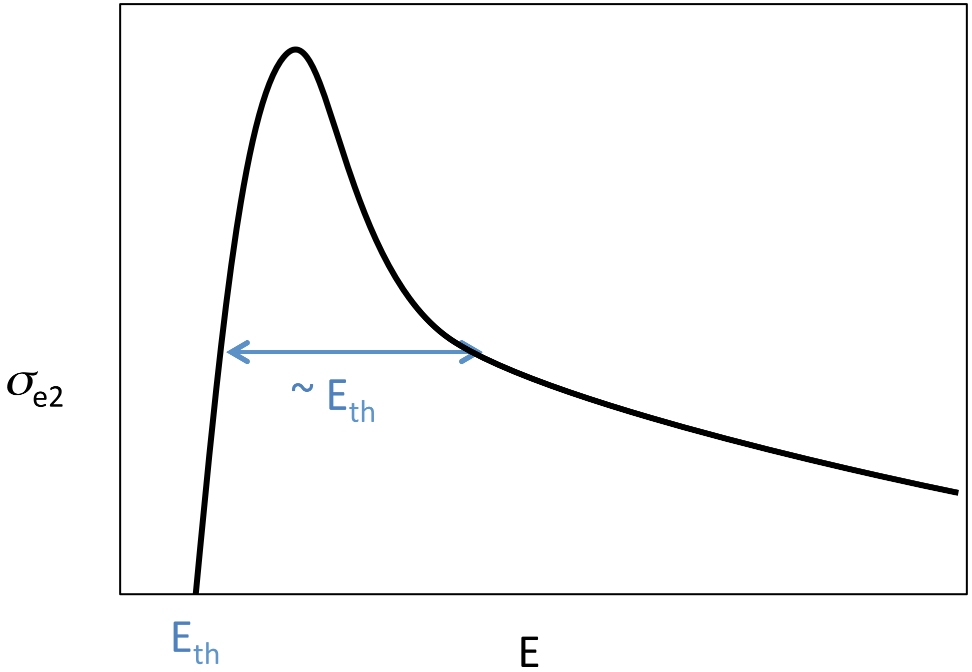
Pump rate

$$R_p = \left(\frac{dN_2}{dt}\right)_p = \sigma_{e2} N_t N_e v$$

- σ_{e2}: Electronic excitation cross-section
  - Dependent on electron energy E, as $\sigma_{e2}(E)$
- N_t: Total species population
- N_e: Electron density
- v: Electron velocity
  - Follow a distribution function $f(E)$

$$R_p = N_t N_e \langle v\sigma_{e2} \rangle$$

σ_{e2}



Width of the curve is much broader compared to optical excitation.

f(E): Electron energy distribution

- $\boldsymbol{v}=\boldsymbol{v}_{\mathrm{th}}+\boldsymbol{v}_{\mathrm{drift}}$

Electric-field-induced velocity

Thermal velocity: random, temperature-dependent, <v>

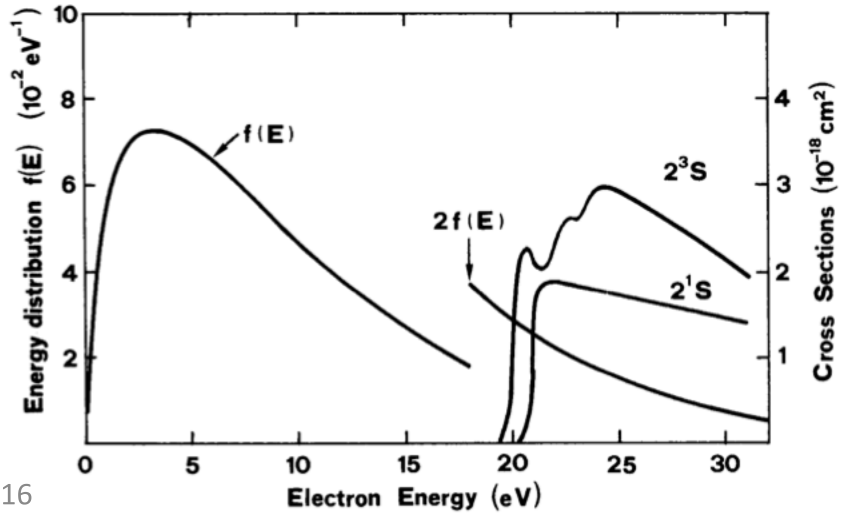
$v_{\mathrm{drift}}/v_{\mathrm{th}} \approx 0.01$

- *f*(E): **Maxwell-Boltzmann distribution**

$$f(E) = \frac{2}{\sqrt{\pi} k T_e} \sqrt{\frac{E}{k T_e}} \exp\left(-\frac{E}{k T_e}\right)$$

T_e: Electron temp.

$T_e \rightarrow f(E) \rightarrow v_{th} \rightarrow \langle v \rangle$



He-Ne:

- f(E):Maxwellian (mean electron energy of 10 eV);
- σ_{e2}: $1^1S \rightarrow 2^1S$; $1^1S \rightarrow 2^3S$ transitions of He;
- High electron energy; inefficient

Balance conditions

T_e is related to electrical field and pressure $T_e = f\left(\frac{\mathcal{E}}{p}\right)$

- **Energy-balance condition:**
  Energy loss by electron collisions = Energy supplied to electrons by E field
- **Momentum-balance condition:**
  Momentum should be conserved with a collision

T_e is related to gas pressure and tube radius $f(T_e) = \frac{Const.}{(pR)^2}$

- **Ionization balance condition:**

  Ionization $\rightleftarrows$ Electron-ion recombination at tube walls

Scaling law of gas laser

$$T_e = f\left(\frac{\mathcal{E}}{p}\right) \qquad f(T_e) = \frac{Const.}{(pR)^2}$$

Optimum T_e (for achieving max R_p) can be obtained with various combinations of R, p and ε.

Example: R↓ → p↑ → $\mathcal{E}$ ↑

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