II2202: Research Methodology and Scientific Writing 2012

Quantitative Research and Data Collection Methods

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Goals!

- The ultimate goal is an understanding of how to collect, analyze and use data in an experiment. Specifically:
- The ROLE of data in a scientific study. What it really is for. It is far more important than you may think.
- PLANNING to get the data you need. What kind of data, and how much is enough.
- How to MEASURE data. How to obtain the data you need.
- How to express the SIGNIFICANCE of your data. Showing what your data means.
- You should be able to exercise these skills as an independent engineer, scientist, entrepreneur or consultant.

We will be talking about topics in Statistics

- This is not a replacement for a full course in Probability or Statistics.
- However, some basic and important concepts from Probability and Statistics will be covered that are applicable to science in general.
- They will be enough to get you started on meaningful data analysis.
- You will want to supplement them with more concepts that represent standard analysis tools in your discipline area.

The idea is: **Let the Data Speak!**

The whole point of data is to *quantitatively* show the value of something.

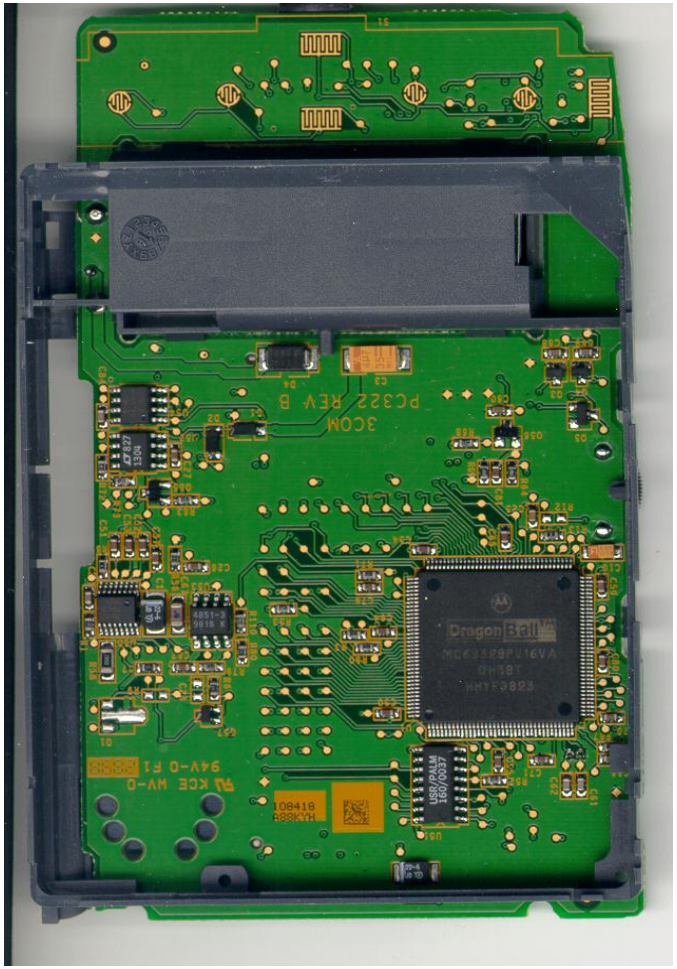
- *Show* how well something works with minimal ambiguity.
- Accurately *predict* how well something can or will work.
- Allow people to *verify* your work by re-doing it.
- *Resolve* selection criteria. Which solution is better based on what?

Other, no less valuable uses:

- Know the preliminary value of a new idea quickly and clearly.
- Establish your credibility. That you know what you are talking about.
- To teach. To pass on knowledge of benefit to others.
- To communicate in a way that others can understand.
- Remove the non-useful elements from technical decisions.

Example: Performance & money. When the data spoke.

It is easy to make a bad design decision if you don't take data into account



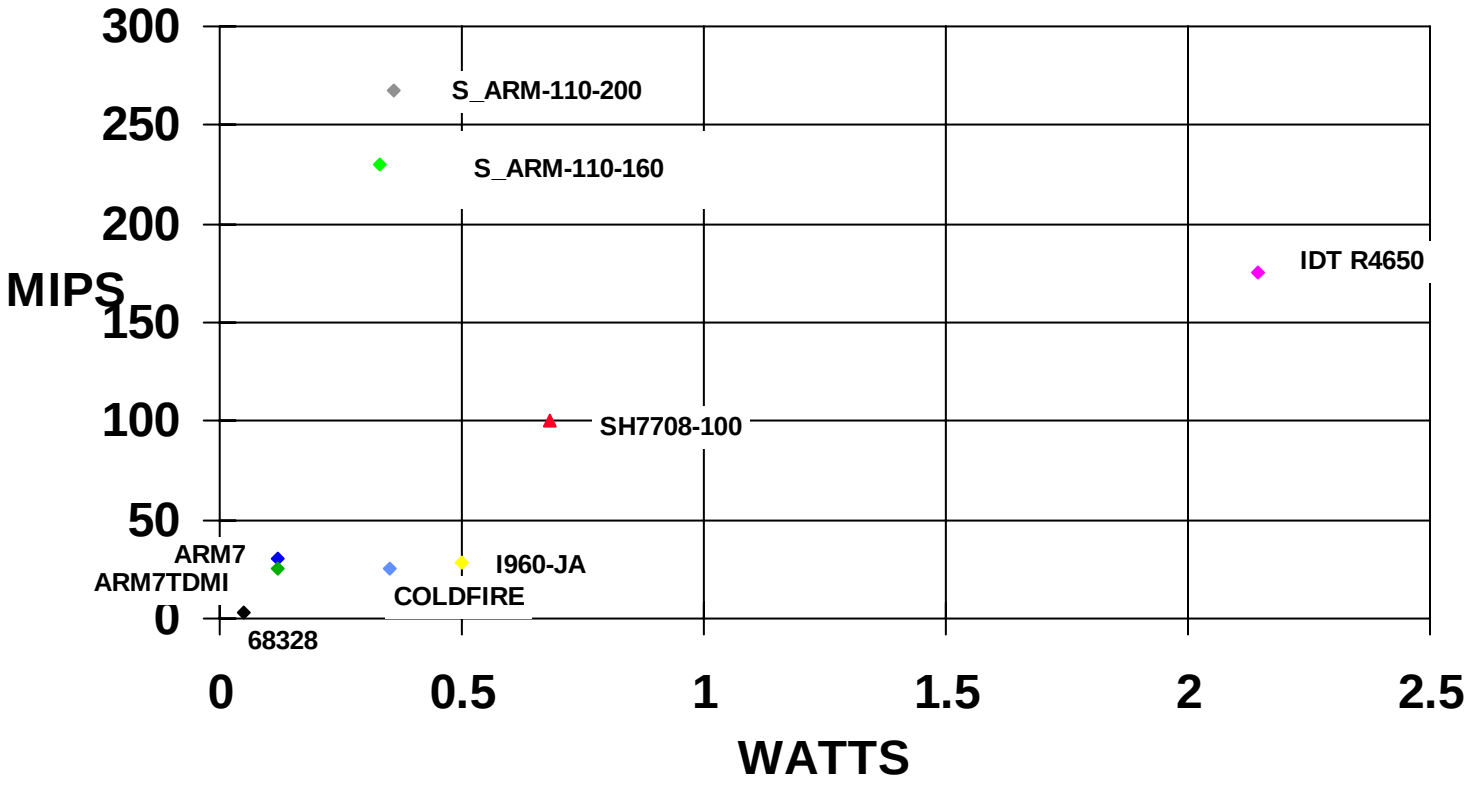
The Palm Pilot III
Released 1998, cost $400 USD

It's mostly one big processor.
(It's a Motorola 68328 'Coldfire')

Why did they choose it?
Why didn't they use an ARM core?

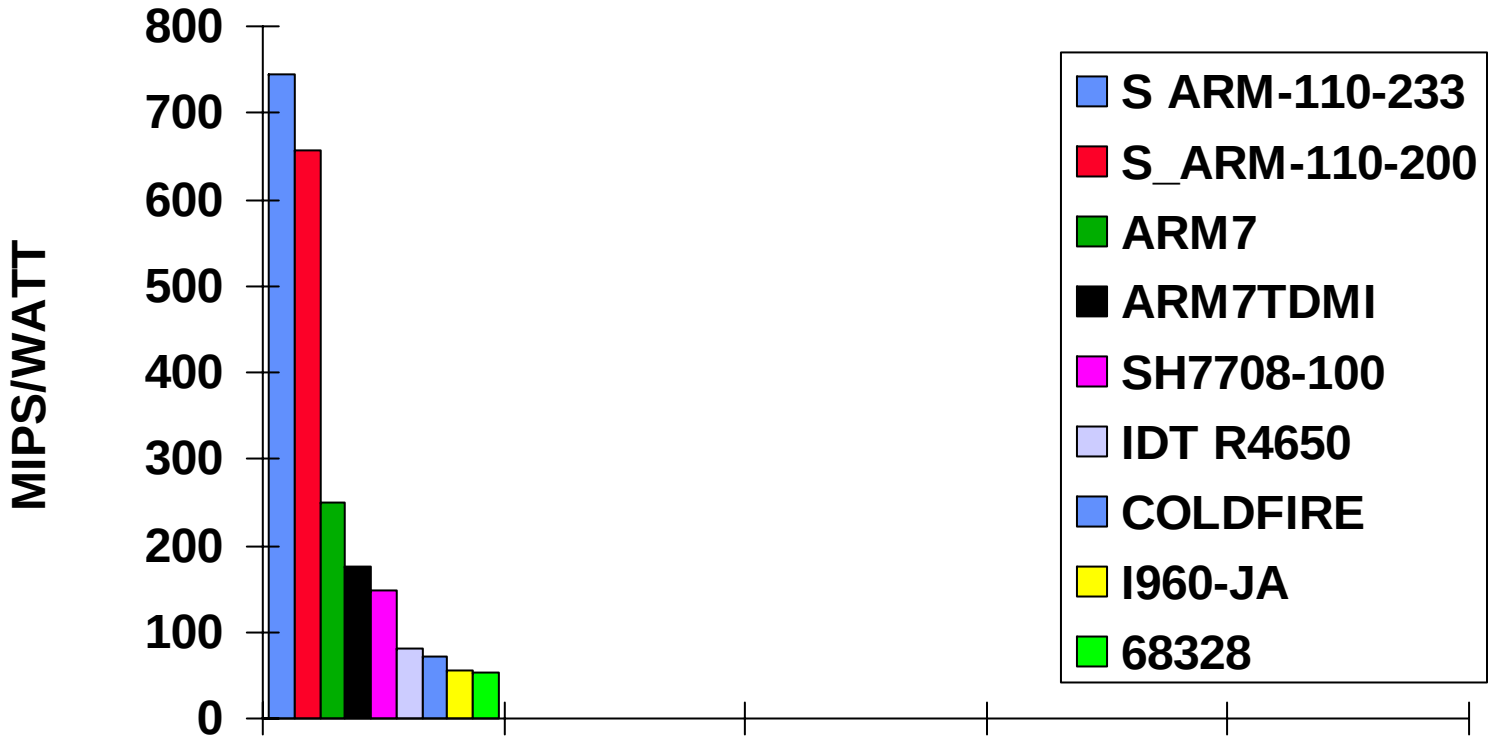
‘MIP’s and Watts

They are both measures of performance. Which do you choose?
Both are driven by a product specification. Want high MIPs and low Watts.



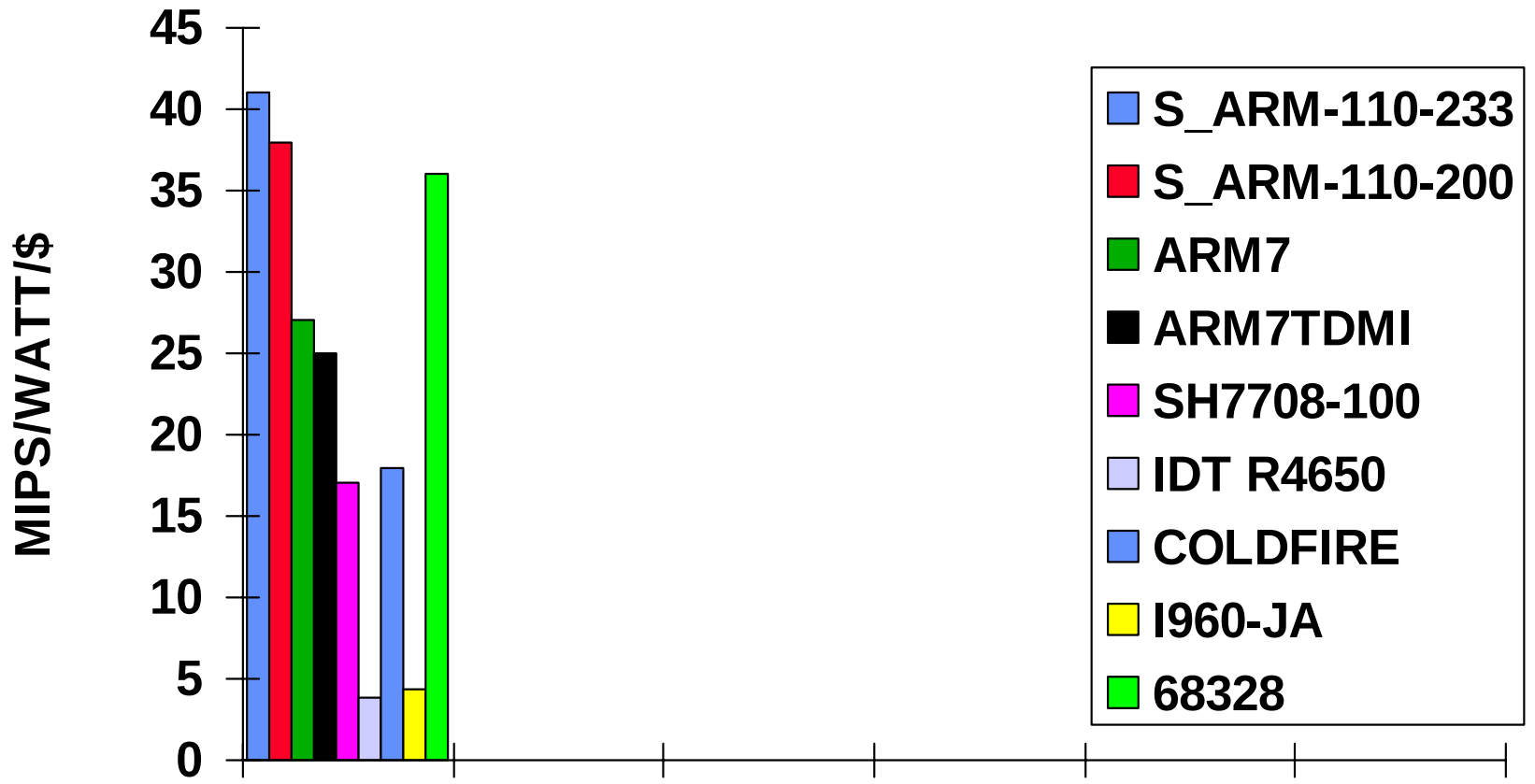
By themselves, MIPS or Watts is not enough for evaluation

Express the data differently: MIPs/Watt



Based on this, we might decide a product based on a StrongARM 110-233 is the right decision. Good performance, but a lot of power still.

Now, include data for cost: (MIPs/Watt)/USD



Very interesting! Turns out the Palm III did choose right.
That was the right choice at the time based on Performance/Watt/Money.

PLANNING, or determining what data you need

Planning is very important. It determines what the design of your study and experiments will be. Wrong experiments, then useless data.

- WHAT is the question you are trying to answer? In other words, what is your *problem statement*?

Examples:

1. "Can a RFID system be designed for a parking solution where the RFID tag can be read within 0.5 seconds when held 4cm or less from the gate reader?" (A technical research statement.)
2. "Would being charged tuition money change a prospective students decision to enroll at the KTH?" (A social research statement.)
3. "Can a personal navigation system be built that can show the position of a person to within 2cm of their actual position?" (Another technical research statement.)

Other points about problem statements

- A problem statement needs to be clearly stated right from the start. If it isn't, nothing else in the document will make any sense.

  "*If you don't know where you are going, then any path will take you there*." (One of my former bosses said this, and it means that if you don't know what you are doing, then little of any value results.)

- Once you state your problem, be sure that your study gives *conclusive* results. Lots of raw data often is not enough to be conclusive.

- Results can be negative! That can be a very useful answer.

- There has to be a clear relationship between the problem statement and your study or experiments. Don't answer the wrong problem!

Determining the data you need and the nature of numeric data

Seems easy. Build it, measure it, done! (No, it doesn't work that way.)

Your data has to match your problem statement needs.

- What *resolution* is necessary for your data?
- What *format* should your data be in?
- How do you actually measure, or *sample* the data?
- *How much* data do you need?
- How do you know your data is not *biased*, or a function of something not intended to be measured (noise)?

You answer these as part of your experimental *planning*.

Depending on your field of study, there may be other considerations.

BAD DATA HAS NO SIGNIFICANCE!

Resolution of *Digital* numbers

Resolution relates to the smallest difference in “true” value that a data number can represent. For digital numbers, resolution is reflected in the value of a Least Significant Bit (LSB). For example:

- An 8 bit byte can only resolve something to within one unit out of 256. (That isn’t very much, only 256 LSBs.)
- A 16 bit representation can only resolve something to within one unit out of 65536.
- A 32 bit representation resolves something to within one unit out of 4294967296. A so forth….

- The more LSBs a representation has, then the more resolution you get.

- HOWEVER, this says nothing at all about how actual data is mapped to these representations! You need to be careful!

Representation of numbers

How you choose to create numbers using some digital representation is up to you. Just be sure the representation is right!

Common examples, assuming binary (x_i can be 0 or 1):

- Integers:

$$X = \sum_{i=0}^{n-1} x_i 2^i$$

- Signed 2s complement:

$$\text{if MSB} = 0 \quad X = \sum_{i=0}^{n-2} x_i 2^i$$

$$\text{else} \quad X = -\left(\left(\sum_{i=0}^{n-2} (\sim x_i) 2^i\right) + 1\right)$$

Where MSB = Most Significant Bit and ~ = bitwise complement.

Representation of numbers

More common examples, assuming binary:

- Fixed Point: $\underbrace{(x_{k-1}x_{k-2}\cdots x_1x_0}_{\text{integral part}} \underset{\text{D.P.}}{\bullet} \underbrace{x_{-1}x_{-2}\cdots x_{-m})}_{\text{fractional part}}$
- or: $X = \sum_{i=-m}^{k-1} x_i 2^i$
- The position of the decimal point is *fixed* in this case. You put it wherever you want based on the resolution you need.
- Floating point numbers also use 2 parts: a Mantissa and Exponent:

$$F = M \cdot \beta^E \quad \text{where M may have 23 bits and E may have 8 bits}$$

- An IEEE 754 floating point number can resolve something to within $1x10^{-95}$ out of + - $9.999999x10^{96}$

Number formats and experiments

- Programming languages and data analysis tools like Excel or Matlab have predefined number formats to represent data.
- Use them. You don’t need to make up your own, but you can if you want or need to.
- When using anything predefined, be sure your number formats will accommodate your measurement range and your required resolution.
- Don’t state high or excessive resolution if it is meaningless with respect to your data.

Here are some examples.

Example: Range and resolution

Suppose you want to measure temperature. You want:

Measurement range: From -40°C to 100°C.
Resolution: 0.5°C

To represent this, what should we use?

To find out, determine the number of LSBs you need:

$$100 - (-40) \Big/ 0.5 = 280 \text{ LSBs}$$

This means you need a data representation having at least:

$$\left(\frac{\log(280)}{\log(2)} \right) = 8.12 \text{ bits}$$

Need more than an 8 bit byte.
A 16 bit representation would be OK.

Example: Excessive resolution

Suppose you read 9 temperature values, and they are:

22, 26, 25, 23.5, 22, 24.5, 27, 26.5, 25

Now suppose you represent these as floating point numbers and you compute the average temperature to obtain:

(22 + 26 + 25 + 23.5 + 22 + 24.5 + 27 + 26.5 + 25) / 9 = 24.611111111111

What does this mean? Not much! Just because your number format can represent a high degree of resolution, that doesn't mean your data does!

Our temperature sensor can only resolve 0.5 degree. So the average of our temperature data readings can imply no higher resolution than that.

Here we can only say that the average temperature is 24.5 degrees.

Getting back to meaningful Data

Now that we know how to represent data with the right resolution, we need to look at how we show what the data really means.

- What is a *measurement*? What is meant by that?
- What is the *accuracy* of your measurements?
- What is the *precision* of your measurements?
- How do these reflect on the quality or the *confidence* one can have in your data?
- How do you make them better, if you need to?

Measurements and other vocabulary

Talking about experiments, data and measurements can be very confusing. We need a standard vocabulary to use.

- An *EXPERIMENT* is a collection of related measurements.
- A *MEASUREMENT* is a data point. Your collection of measurements is what makes up your experimental data.
- Sometimes, people call a measurement a *STATISTIC*.
- A measurement is made by taking *SAMPLES* of something. For example a voltage reading, or someone's opinion. You determine how many samples per measurement you need.
- Note that the *number of samples* you take per measurement is often NOT the same thing as the *number of measurements* you have in an experiment!

Accuracy and Precision

Given a “true” value to measure:

Accuracy relates to the difference between your measurement and the “true” value.

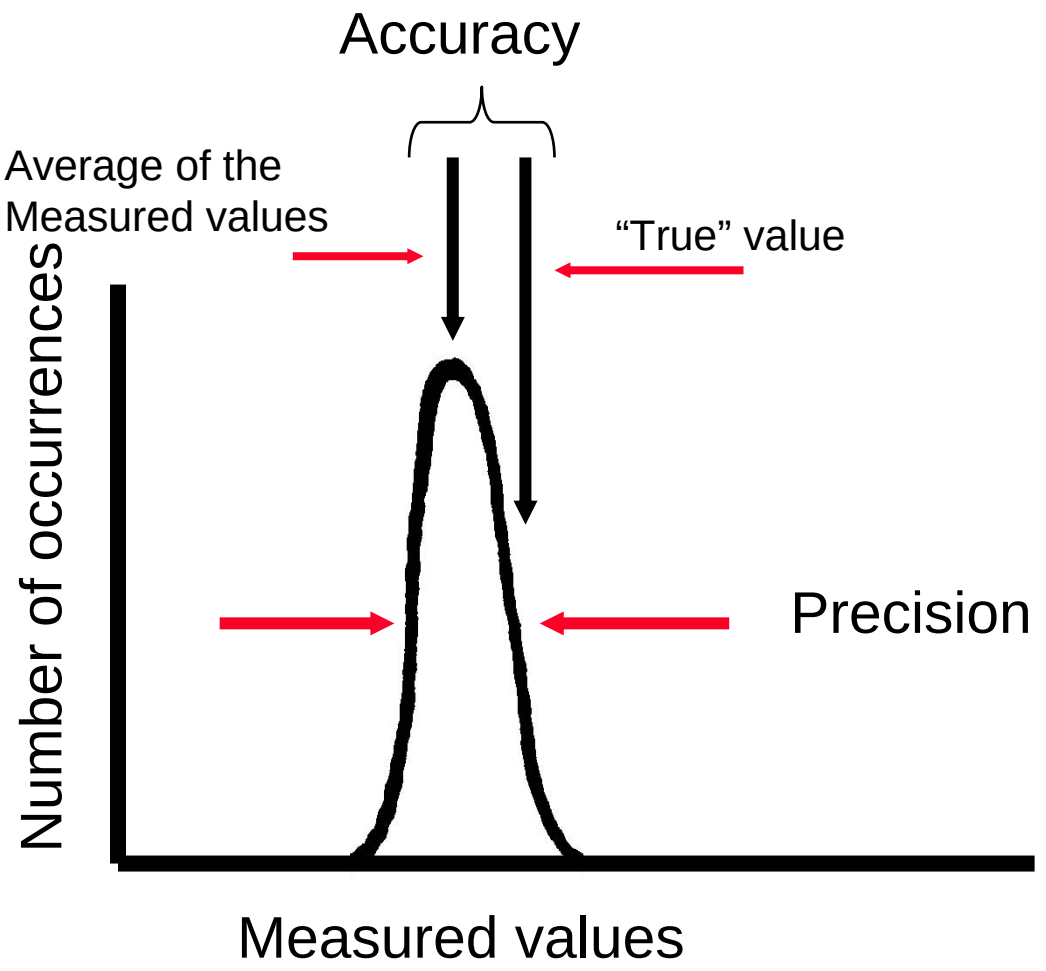
Precision relates to how repeatable your measurements are. It’s possible to be very precise, but not very accurate. It’s also possible for a group of measurements taken together to be quite accurate, but not very precise.

Example: Accuracy and Precision

Accuracy: Is the difference between the "true" value and the average of your actual measurements of the "true" value. Perfect accuracy would result in the average of the actual measurements of the "true" value being exactly the same as the "true" value.

Precision: A measure of the value spread of your actual measurements of the "true" value. Perfect precision would have a spread of zero. The wider the spread, the worse the precision.

Note that it is possible to have good accuracy with bad precision. Also it is possible to have good precision with bad accuracy.



Measurements and meaning

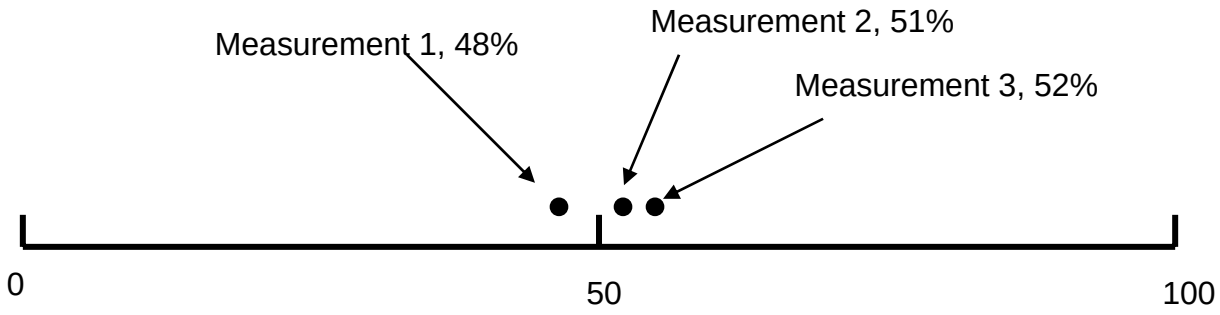
- The reason for making measurements or collecting data is to be able to answer some research question.
- But, how good is your answer? That depends on your data.
- For example, how accurate must data be to be good?
- What about imprecise data? Is that always bad?
- How does one express how good the data is?
- Good data is meaningful data. The key to expressing meaning has its roots in accuracy and precision.

A hypothetical experiment

"What percentage of KTH students are in favor of charging tuition for taking KTH courses?"

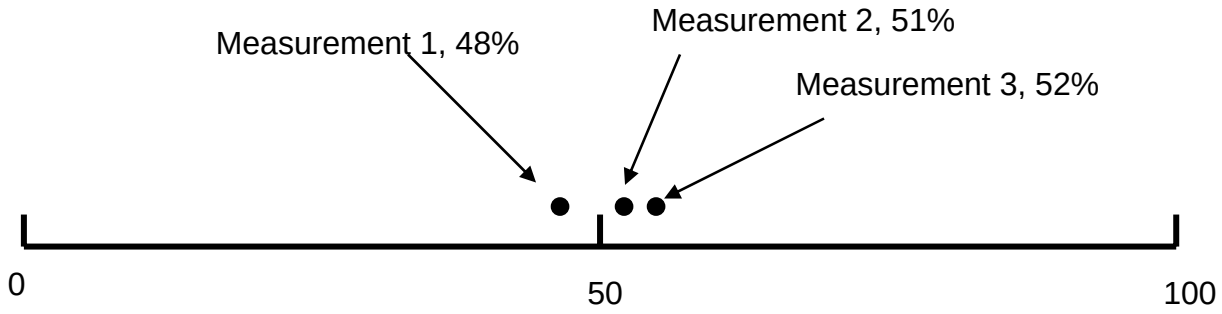
- There are about 14,500 KTH students total.
- If you could ask them all you would get the "true" answer.
- But, finding and asking them all is not really practical.

- So instead, you go to the list of KTH students and randomly select 100 of them and ask them. You do this 3 times to get 3 measurements.



Percentage of students who approve the charging of tuition

Estimating the “true” value



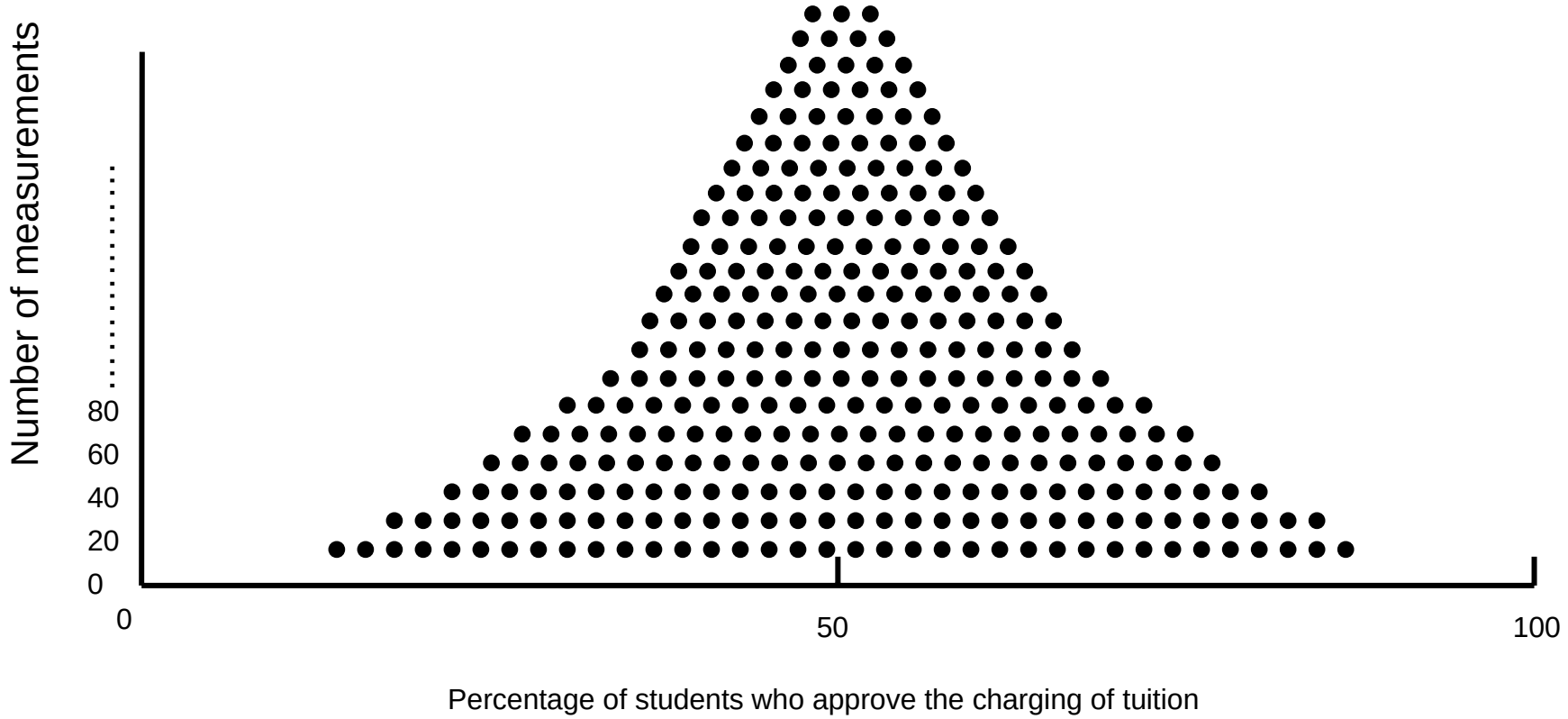
Percentage of students who approve the charging of tuition

- Each random sample of 100 students give us an estimate of the true value.
- But we have 3 estimates here, and they are all different!
- They can’t all be right, so clearly we do not have perfect accuracy.
- They are all different, so we don’t have perfect precision either.

- How can we use this to estimate what the “true” value really is? To see that, let’s get a lot more 100 student samples and plot them.

A lot more samples

As we take more measurements, the data starts to show meaningful things. The values with the highest number of measurements is probably nearer to the “true” value. The shape of the data is also interesting.



Mean and standard deviation

The mean is the average of your measurements. It is an estimation of the "true" value you are trying to find out. It reflects the accuracy of the data.

The standard deviation is the average of how the measurements differ from the mean. It reflects the precision of the data.

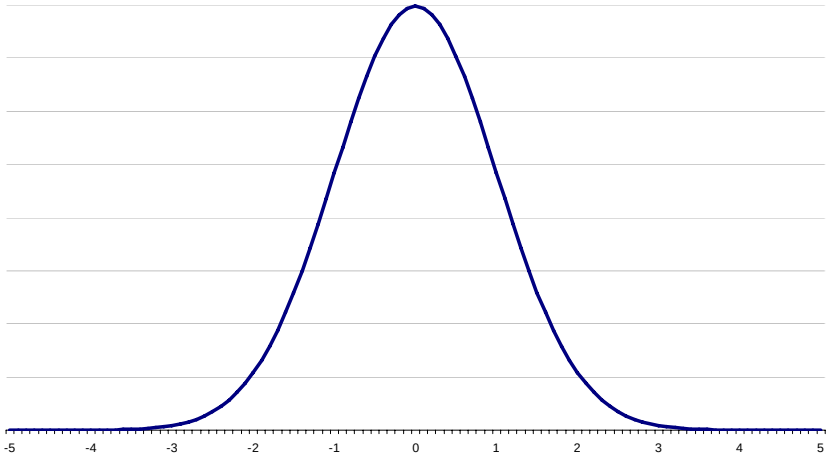
$$\mu = \frac{1}{N}\sum_{i=0}^{N-1} k_i$$

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=0}^{N-1}(k_i - \mu)^2}$$

N = total number of *measurements*
k = *measurement* value

In this Gaussian Distribution

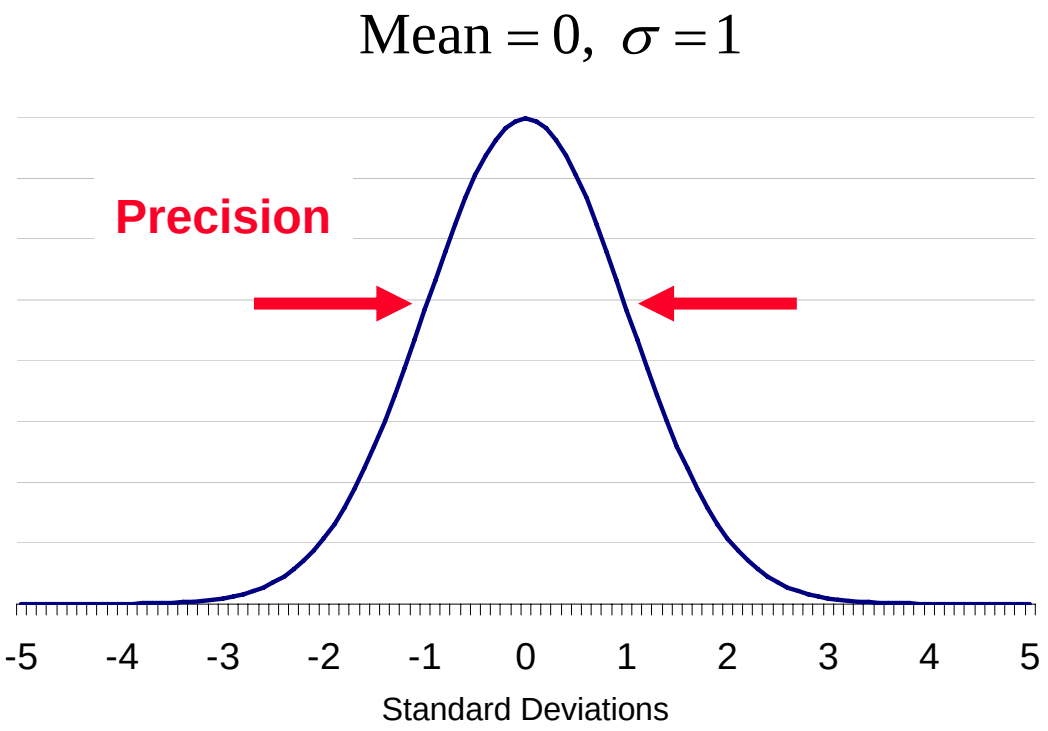
$\text{Mean} = 0,\ \sigma = 1$



Note that N is <u>not</u> the number of samples per measurement!

Precision of the measured data

You can see now the relationship between standard deviation and precision. The larger the standard deviation, the worse the precision.



The reason this is important is that it allows you to predict how much confidence you have that your data is within a certain distance of the “true” value. In other words, it is related to how good your data is.

Standard Deviation and measurement error

- The standard deviation is the average of how the measurements differ from the mean. That amount is ONE standard deviation.
- Lets say in our experiment about the question of tuition that we get the result that the mean=0.5 and the SD = 0.025.
- In other words, the experiment estimates that 50% of the students are in favor of the KTH charging tuition. One SD is 5% of this. Our estimate is not precise, and the amount that it is off averages to 5%.
- This 5% that is our one SD is also called our *standard error*.

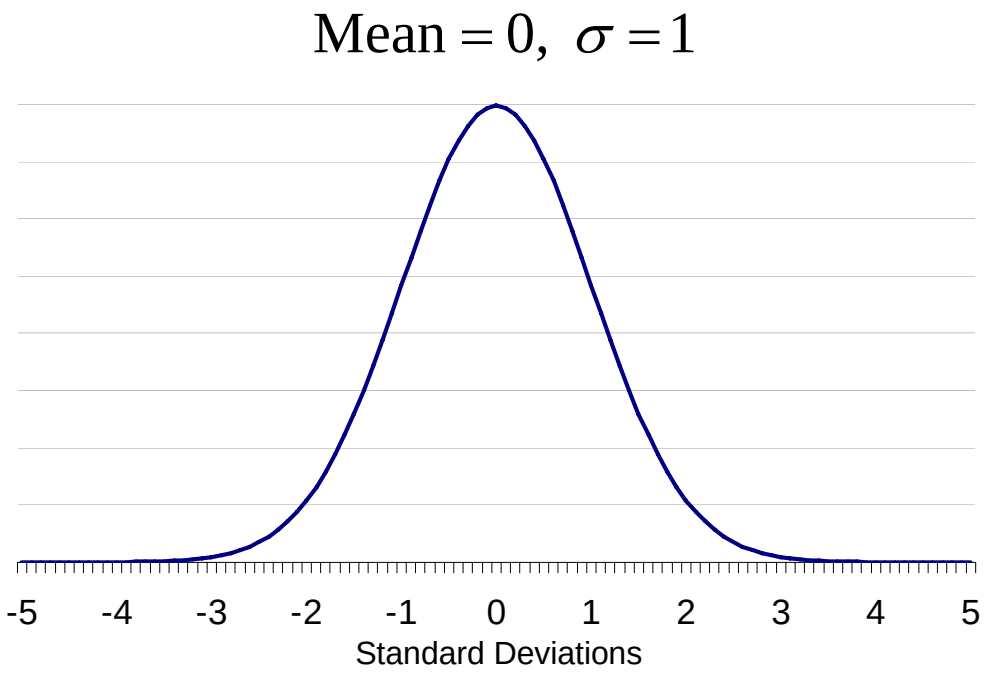
Standard error

- Standard error gives you a measure of how good your data is because it tells you how reliably a single measurement will estimate the “true” value.
- In our experiment, it says that one measurement consisting of 100 randomly sampled students will on average be wrong by 5% of the experiment mean value.
- This is OK, but it isn’t very useful. We don’t want to know how wrong the experiment will be.
- We want to know how confident we can be that the experimental data reflects the “true” value, and by how much.
- For this, we can take advantage of the properties of Gaussian distributions to help us.

Properties of normal distributions

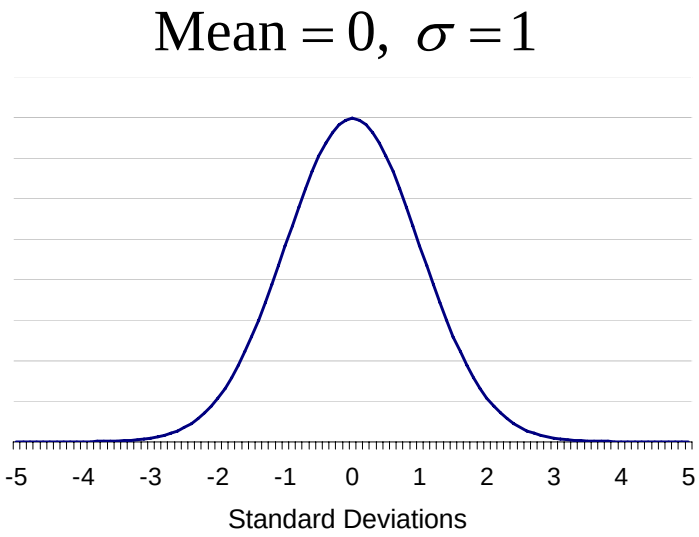
- Normal distributions have some known properties that have been determined by probability theory.
- A normal curve (or Gaussian distribution) is telling you the probability of where a measurement value could be in the context of the study.
- The area under parts of the normal curve determines this.
- To see this, let's start with a normalized Gaussian distribution. It is normalized in that the indicated mean is at zero.

Properties of normal distributions



- The probability that a measurement value for this experiment will exist anywhere under the normal curve is 1.
- The area under one SD is also consistent in a normal curve. This area is equal to the probability that a measurement will fall within one SD away from the mean or “true value”.
- To determine that, we need to know the area of the curve between the mean and one SD.

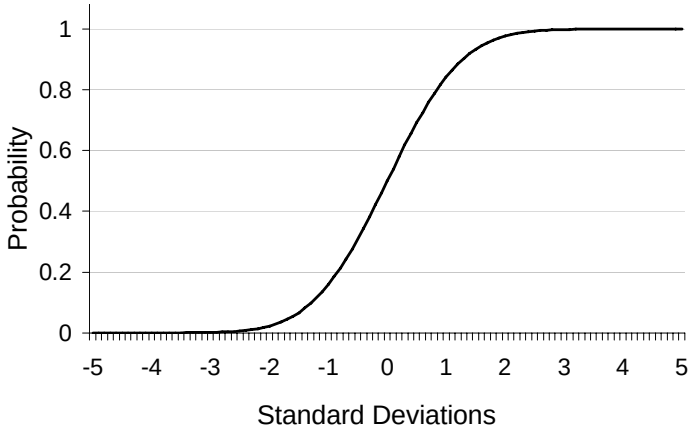
You can compute areas, or look them up



$$F(x) = \frac{1}{\sqrt{2\pi\sigma}} \int_{-\infty}^{\infty} \exp\left[\frac{-(u-\bar{X})^2}{2\sigma^2}\right] du$$

Normalizing $\bar{X} = 0$ and $\sigma = 1$

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{X} e^{-u^2/2} du$$



This is called the Cumulative Distribution Function (CDF) of the normal curve. From this you can take off numbers that give you the probability of being in some range of the normal curve.

Using the CDF to determine probability

The probability of a measurement value being within 1 SD of the mean:

$$P = \Phi(1) - \Phi(0)$$
$$= 0.8413 - 0.5$$
$$= .3413 \text{ or } 34\%$$

The probability of a measurement value being within +-1 SD (2 SDs) of the mean:

$$P = \Phi(1) - \Phi(-1)$$
$$= 0.8413 - 0.1587$$
$$= .6826 \text{ or } 68\%$$

X	$\Phi(x)$	X	$\Phi(x)$
-3.4	0.0003	0	0.5000
-3.3	0.0005	0.1	0.5398
-3.2	0.0007	0.2	0.5793
-3.1	0.0010	0.3	0.6179
-3	0.0013	0.4	0.6554
-2.9	0.0019	0.5	0.6915
-2.8	0.0026	0.6	0.7257
-2.7	0.0035	0.7	0.7580
-2.6	0.0047	0.8	0.7881
-2.5	0.0062	0.9	0.8159
-2.4	0.0082	1	0.8413
-2.3	0.0107	1.1	0.8643
-2.2	0.0139	1.2	0.8849
-2.1	0.0179	1.3	0.9032
-2	0.0228	1.4	0.9192
-1.9	0.0287	1.5	0.9332
-1.8	0.0359	1.6	0.9452
-1.7	0.0446	1.7	0.9554
-1.6	0.0548	1.8	0.9641
-1.5	0.0668	1.9	0.9713
-1.4	0.0808	2	0.9772
-1.3	0.0968	2.1	0.9821
-1.2	0.1151	2.2	0.9861
-1.1	0.1357	2.3	0.9893
-1	0.1587	2.4	0.9918
-0.9	0.1841	2.5	0.9938
-0.8	0.2119	2.6	0.9953
-0.7	0.2420	2.7	0.9965
-0.6	0.2743	2.8	0.9974
-0.5	0.3085	2.9	0.9981
-0.4	0.3446	3	0.9987
-0.3	0.3821	3.1	0.9990
-0.2	0.4207	3.2	0.9993
-0.1	0.4602	3.3	0.9995
0	0.5000	3.4	0.9997

A few more useful Gaussian properties

- The probability of a measurement value being within +-1 SD of the true value is 68%.
- The probability of a measurement value being within +- 2 SD of the true value is 95%.
- The probability of a measurement values being within +- 3 SD of the true value is 99.9%
- This is true of ALL Gaussian distributions!

Confidence

- Now, we can say something useful about our data. We can say:

"I am 95% confident that that between 40% and 60% (+- 2 SD) of students approve of the idea of charging tuition."

- This is useful and valuable because we have specified two new components:

- *Confidence interval*: A range of values within which a measurement value is estimated to be. In our case our confidence interval is within 2 SD (plus or minus) or +- (2 X 5%) of the true value.

- *Confidence level*: The estimated probability that a measurement value is within the stated confidence interval. In our case our confidence level is .95 or 95%.

Increasing the precision

- In our example, I claimed:

  "I am 95% confident that that between 40% and 60% (+- 2 SD) of students approve of the idea of charging tuition."

- Sounds nice, but 40% to 60% is not all that precise. How can I get better precision?

- Recall that 1 SD in our study is 5%. It was determined by solving the equation for standard deviation.

- Let's look again at the formula and see what makes it work.

Better precision

$$\sigma = \sqrt{\frac{1}{N-1}\sum_{i=0}^{N-1}(k_i - \mu)^2}$$

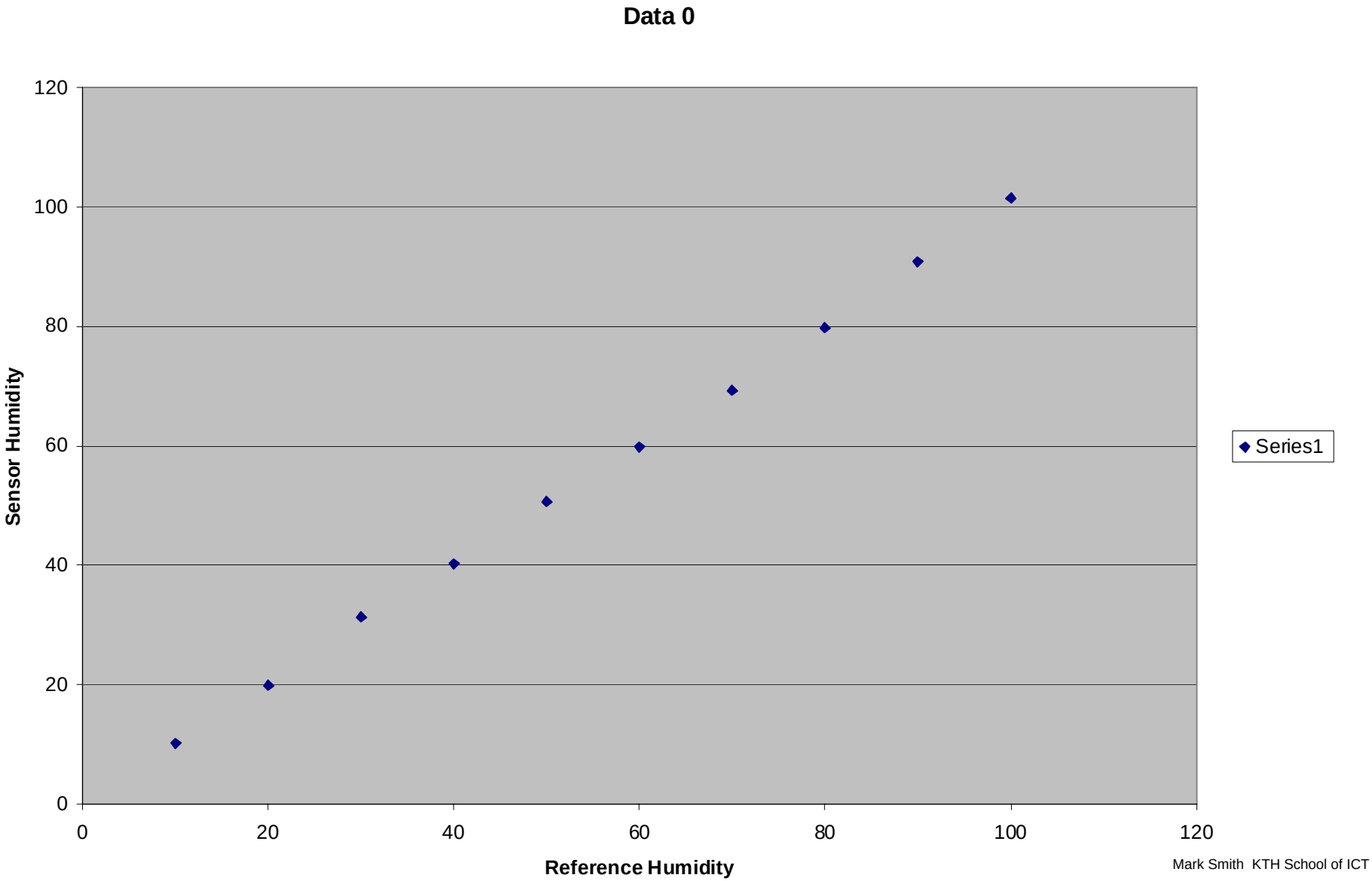
- u is the estimation of the true value.
- N is the number of measurements. It looks like increasing N may help, but not really if the difference between the k and u values are big. The key is the relationship between k and u.
- k is a measurement. The closer its value is to u then the smaller the SD.
- Remember that k is itself a mean. It is the mean of the 100 samples used in each measurement!
- So, by increasing the sample size, k will approach the value of u!
- When that happens, your confidence interval can get smaller. That's good.

Better precision

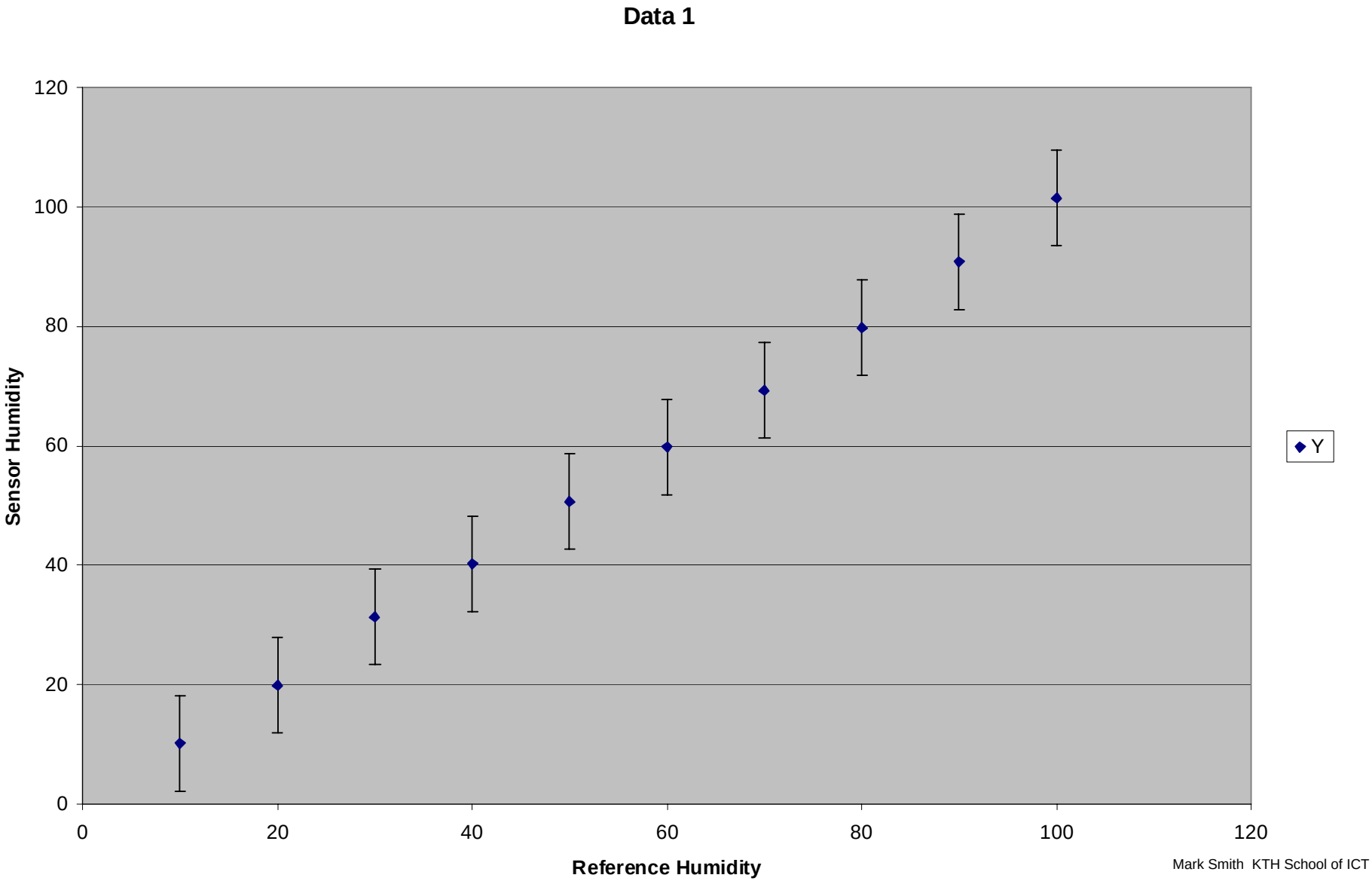
$$k = \frac{1}{S}\sum_{i=0}^{S-1} p_i \text{ and } \mu = \frac{1}{N}\sum_{i=0}^{N-1} k_i$$

- S is the sample size, and N is the number of measurements.
- p is the value given by each person who is sampled in the study.
- As S gets larger and larger, k starts to approach u, the true mean.
- That means that as S gets bigger, the difference between u and each measurement (k_i) gets less.
- Standard deviation goes down, and precision goes up.
- Choose your sample size for the precision/error tolerance you need.

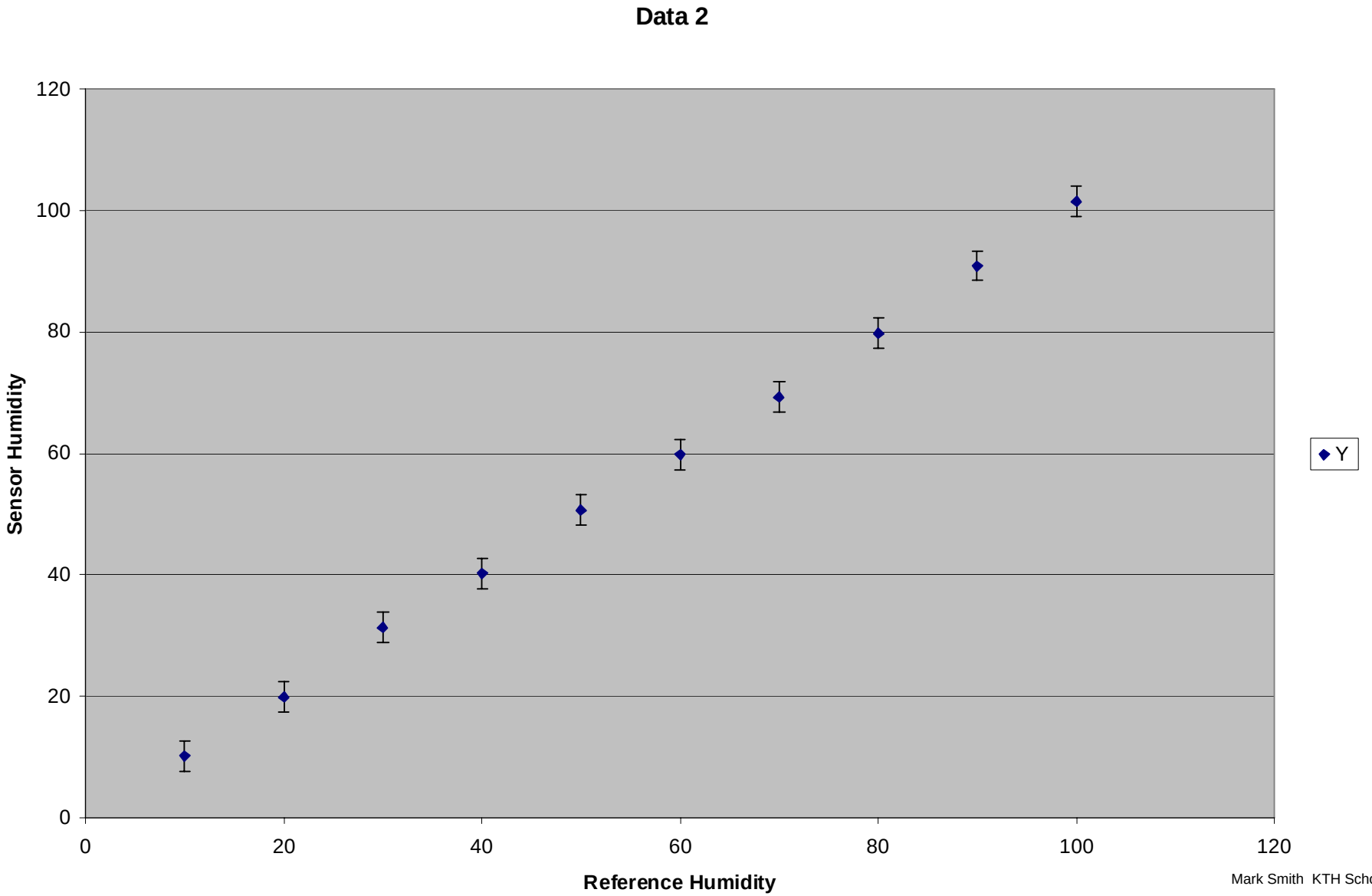
Example use: A Humidity Sensor



This says a lot more
(and shows a problem too!)

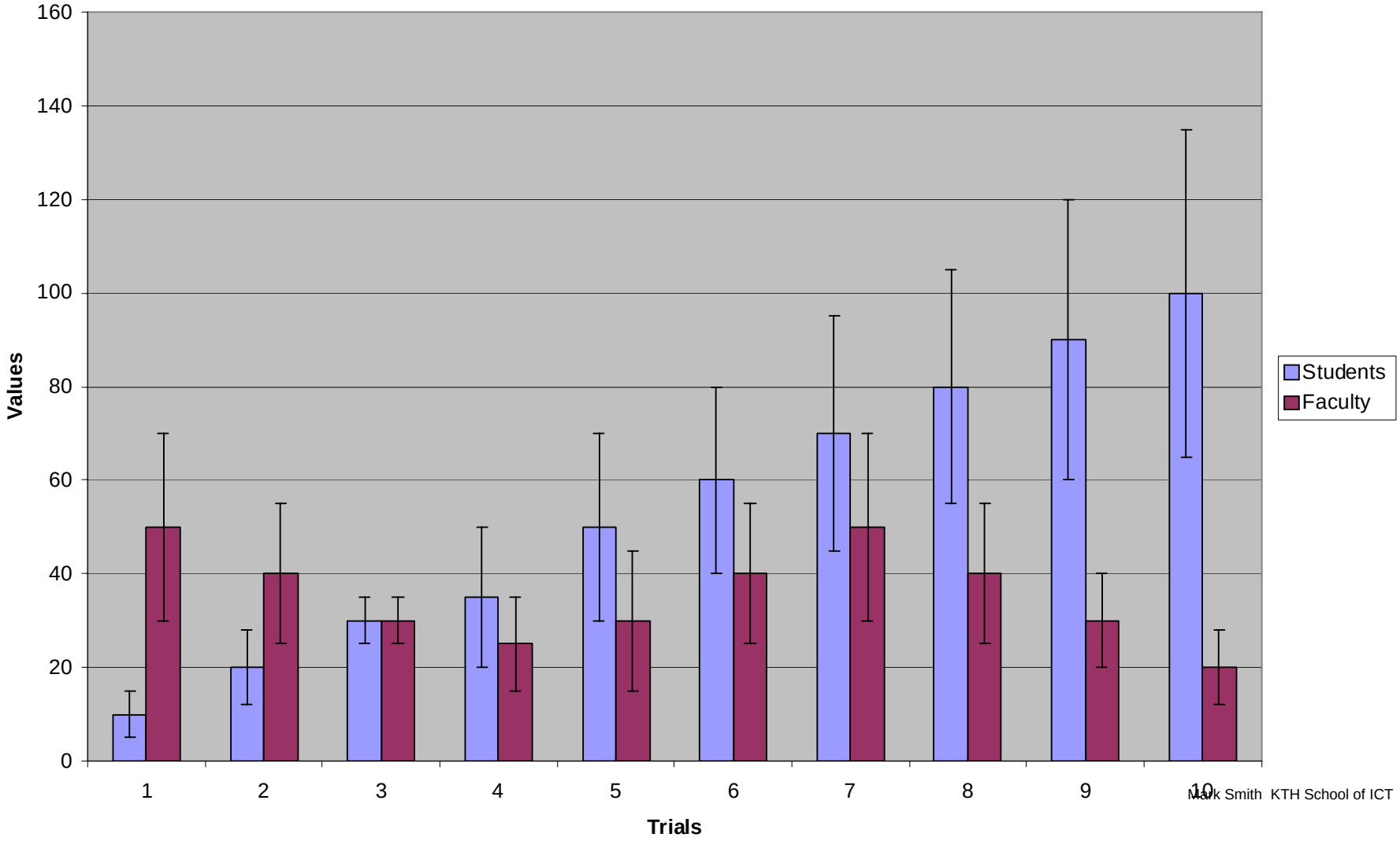


With lower Standard Error
(more samples per measurement)



Example: Histograms

Where can you see evidence of confidence of a significant difference between “students” and “faculty”? The error bars can be for any confidence level you want.



More tools

- We have seen how resolution, measurements and precision can work together to allow you to interpret the meaning of your data.
- This is done by using Gaussian distribution properties to help analyze how your data varies with respect to the “true value”.
- They also allow you to express where you are confident about the meaning of your data (confidence interval) and the degree to which you are confident (confidence level). This establishes the significance of your data.
- There are many other statistical tools that allow you to analyze how your data varies, and establish its significance. For example one and two way Analysis of Variance (ANOVA).
- Find them. Use them. Excel has many of these tools built in.

Bias in measurements

- A study that is supposed to evaluate something needs to be independent of things that could influence or change the data in some way.

- There are lots of things that can influence data, but unless you are studying those things you need to design your experiments in a way where they have either EQUAL effect on all of your data.

- Any influence that is not part of your study, and does not effect your data equally is a ***bias***.

- Your data and now the experiment now reflects the bias. Your results will be wrong.

Bias

Consider our hypothetical student study about tuition. What if our sampling had not been completely random?

- What if we had chosen students mostly from the EU?
- What if we had chosen students mostly from non-EU countries?
- Bias is often hard to see. You don't introduce a bias on purpose, but mistakes in planning can introduce large bias.
- For example, what if you only chose students from those living in student housing? You may not realize the bias you introduce.

A few sources of Bias

- Where you get or sample test subjects.
- Lab equipment and measuring devices. Always the same? Are they properly calibrated?
- Experimental conditions, such as where experiments are done or how questions are asked.
- Parts, chemicals, supplies. Same or different manufacturers?
- Experimental procedures or processes. Doing things identically.

There are many more.

Notes about equipment and testing

Lab equipment and how you use it can be a major source of bias or errors. Here are some ideas to improve your data.

- Document every piece of equipment you use. Who made it? What model number is it? Does it have any special options or accessories? What is its resolution? Is it enough?
- Use the same equipment for the entire study. Don't use one set of equipment for one experiment, and different equipment for another experiment unless the experiments have no relation to each other.
- Calibrate your equipment! Is it accurate? Can it really measure a "true value"?
- What is the precision of your test equipment? Are you measuring the variance of your data, or the variance of your test equipment?

Noise and errors

- Be very careful about noise sources. Are you measuring your circuit, the test fixture or the lab environment?

- Identify all possible sources of noise. Try to remove noise:
  - Better environment.
  - Shielding or filtering, for example use batteries instead of AC.
  - Better test fixtures. You may have to design and build one!
  - Better components, for example lower noise amps or ADCs.
  - Be especially careful with reference voltage sources!
  - Is your test equipment affecting your experiment, ie loading your circuit?

- Be sure you document all details regarding noise, or you may not be able to reproduce your test environment.

- Don’t forget to completely document any test fixtures you make!

Process and errors

- The way you perform experiments needs to be consistent across your entire study. If not, you will have bias or errors.
- Hook things up the same way. If possible, use the same equipment (not just the same kind).
- Control things the same way. Use the same parameters, such as numbers of samples and how equipment is adjusted and used.
- Automation is good! Automate how equipment is controlled, and how data is collected. This helps keep things repeatable and consistent.
- Be sure any automation does not introduce any noise, errors or bias on it's own, for example an imprecise clock, or not enough measurement resolution. Test everything using a known standard!