A Linearized Statistical XPM Model for Accurate Q-factor Computation

Marc Ruiz, Luis Velasco, Paolo Monti, and Lena Wosinska

***Abstract*—Non-linear physical layer impairments make the mathematical programming formulation of off-line impairment-aware (IA) Routing and Wavelength Assignment (RWA) problem to be non-linear. To alleviate that, this paper presents for the first time a statistical linear model to compute the Cross Phase Modulation (XPM) noise variance. Exhaustive evaluation reveals that the proposed linear model provides an accurate *Q*-factor estimation. An example of Integer Linear Programming (ILP) formulation integrating the XPM model is also given.**

***Index Terms*—Physical layer impairments, off-line IA-RWA, statistical modeling.**

I. Introduction

In transparent Dense Wavelength Division Multiplexing (DWDM) networks, physical layer impairments (PLI) degrade the signal quality of optical connections (referred to as *lightpaths*) when they traverse the optical fibers and components. PLI can be either *non-linear* or *linear*. Non-linear impairments affect not only each wavelength channel individually, but also cause disturbance and interference among channels traversing the same fiber link. In general, they manifest as signal power fluctuations in long haul links. In contrast, linear impairments do not depend on the signal power and affect each channel individually. The most tangible non-linear effects are Self Phase Modulation (SPM), Cross Phase Modulation (XPM), and Four Wave Mixing (FWM) while the most important linear impairments are fiber attenuation, Amplifier Spontaneous Emission (ASE) noise, Chromatic Dispersion (CD) (or Group Velocity Dispersion (GVD)), and Polarization Mode Dispersion (PMD). The impact of PLI on the transmission quality of a lightpath can be quantified by using the quality factor Q [1]. Eq. (1) shows an expression to estimate the Q-factor:

$$Q = \frac{pen_{eye} \cdot P_{tx}}{pen_{PMD} \cdot \sqrt{\sigma^2_{ASE} + \sigma^2_{XPM} + \sigma^2_{FWM}}}, \quad (1)$$

where P_{tx} denotes the transmitted signal power, pen_{eye} the relative eye closure penalty attributed to SPM/GVD and optical filtering, and pen_{PMD} is the power penalty due to PMD while σ^2_{ASE}, σ^2_{XPM}, and σ^2_{FWM} denote the electrical variance of ASE noise, XPM, and FWM, respectively.

In order to provide good quality lightpaths in transparent optical networks, PLI information needs to be considered when solving the Routing and Wavelength Assignment (RWA) problem. The impairment aware (IA) and off-line RWA problem has recently received a lot of attention. The Q factor computation within mathematical programming formulations entails non-linear constraints, highly increasing the complexity of the problem. For this reason different approaches in the literature propose to compute the effect of PLI only partially, leading to near optimal solutions [2]- [4]; linear PLIs are commonly included in Integer Linear Programming (ILP) formulations, since they can be pre-computed beforehand, whereas non-linear impairments, as a consequence of the wavelength assignment, have been considered in few works. For instance, the authors in [3] propose an ILP formulation designed for reducing as much as possible the interference between lightpaths, but computing the Q value of each lightpath in a post-optimization process. Other works propose ILP formulations with similar constraints combined with iterative methods, where the Q factor is still computed outside the ILP. Among them, the authors in [4] propose a complex algorithm consisting of four simple ILP formulations. None of these approaches listed so far include a Q-factor computation within their ILP formulations, with no guarantee that an optimal solution is eventually found.

While looking at the nature of physical impairments, it was found that, in the case of 10 Gbps signals with On-Off Keying (OOK) modulation, the value of XPM is dominant over FWM, being XPM variance several times higher than the one of FWM [5], [6]. With this in mind, a statistical and non-linear model for fast and accurate estimation of the XPM noise-like variance was proposed in [6] and it was used for fast computation of the Q-factor for a given lightpath. The model in [6] was specifically designed for dynamic scenarios, so its linearity was not a requirement. This paper goes a step further proposing, for the first time, a statistical linear model to compute the XPM noise-like variance of a lightpath. Exhaustive numerical simulations are performed to validate the XPM model against analytical expressions. Finally, since the proposed model is specifically designed to fit off-line scenarios for the IA-RWA problem, an ILP formulation is detailed for illustrative purposes.

II. XPM model

This section first describes the assumption made for the transmission kind, it provides an analysis of the nature of the XPM variance, and then it details the proposed XPM model.

A. XPM Noise Variance Analysis

Let $G(N, E, W)$ be a graph describing an optical network, where N is the set of nodes, E is the set of fiber links, and W is the set of wavelengths, each one associated with a wavelength channel labeled from 1 to $|W|$. Additionally, let α_e be the number of optical amplifiers in link $e \in E$. The transmission link consists of a sequence of single mode

Manuscript received March 26, 2012. The associate editor coordinating the review of this letter and approving it for publication was J. Wang.

M. Ruiz and L. Velasco are with the Universitat Politècnica de Catalunya (UPC), Barcelona, Spain (e-mail: {mruiz, lvelasco}@ac.upc.edu).

P. Monti and L. Wosinska are with the Royal Institute of Technology (KTH), Kista, Sweden (e-mail: {pmonti, wosinska}@kth.se).

Digital Object Identifier 10.1109/LCOMM.2012.061912.120680

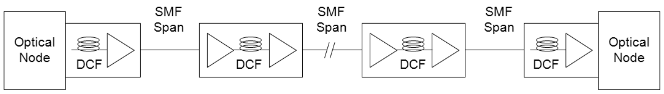

Fig. 1. Transmission link architecture.

fiber (SMF) spans (Fig. 1). Their number varies according to the physical distance. In the current study, the length of each fiber span is assumed to be 80 km. Optical amplifiers are inserted after each fiber span to compensate for the power loss induced by the fiber. A non-resonant dispersion map that utilizes pre- and post- compensation is used, while the accumulated dispersion is also compensated by allocating in-line dispersion compensating fiber (DCF) after each amplifier. The detail of transmission link model used in this work can be found in [1].

Since the XPM noise variance of a lightpath depends on the physical route and the assigned wavelength, let $\sigma^2_{XPM}(r,\lambda)$ represent the value of XPM variance of a lightpath using route r and wavelength λ. Besides, let $\sigma^2_{XPM}(e,\lambda)$ be the XPM noise variance on reference channel λ in link e. As shown in [7], $\sigma^2_{XPM}(r,\lambda)= \sum_{e\in E(r)} \sigma^2_{XPM}(e,\lambda)$, where $E(r)$ is the subset of links in route r. Finally, let $\sigma^2_{XPM}(e,\lambda,i)$ be the XPM noise variance over the reference channel λ as a consequence of channel i, in link e. According to [7], $\sigma^2_{XPM}(e,\lambda)= \sum_{i\in W\setminus\{\lambda\}} \delta_{ei}\cdot\sigma^2_{XPM}(e,\lambda,i)$, where δ_{ei} is equal to 1 if channel i is in use in link e. In summary, each channel being used by an active lightpath adds some interference to the XPM variance of the reference channel independently of the rest of the channels.

This additive property allows considering an alternative way to calculate $\sigma^2_{XPM}(e,\lambda)$ based on the modeling of $\sigma^2_{XPM}(e,\lambda,i)$. In this regard, although each channel in use adds some interference to the XPM variance over the reference channel, this interference decreases with the spectral *distance* between the channels until the gap is too large to have any significant effect. Based on this, it is possible to determine which channels cause a notable interference over the reference one. For example, the work in [6] defines the so-called *channel-interference negligible distance* (η), which means that the channels at a distance greater than η from the reference channel are assumed to add a negligible XPM interference contribution. The notion of η can be used to derive a *restricted* model of $\sigma^2_{XPM}(e,\lambda)$ (2), where ϵ_e represents the error as a result of dismissing those channels at a distance longer than η. From [6], only 3% of error is obtained when η=4.

$$\sigma^2_{XPM}(e,\lambda) = \sum_{\substack{i=max\{1,\lambda-\eta\}\\ i\neq\lambda}}^{\min\{\lambda+\eta,|W|\}} \delta_i(e)\cdot\sigma^2_{XPM}(e,\lambda,i)+\epsilon_e \quad (2)$$

B. A restricted linear XPM model

In order to be able to model $\sigma^2_{XPM}(e,\lambda)$ with a linear function, it is necessary to find a linear expression for $\sigma^2_{XPM}(e,\lambda,i)$ in (2). To this end, we propose a *restricted linear* XPM model to estimate each $\sigma^2_{XPM}(e,\lambda,i)$ value for the channels in the range [λ-η, λ+η]. Each approximation can be denoted as $s^2_{XPM}(\alpha_e,\lambda,i)$, where $\sigma^2_{XPM}(e,\lambda,i)\approx s^2_{XPM}(\alpha_e,\lambda,i)$. The model for estimating $s^2_{XPM}(\alpha_e,\lambda,i)$ is a continuous function expressed in terms of λ that consists in a number of C connected linear segments, each represented by a slope and by two end wavelengths. Eq. (3) formally describes the restricted linear model, where $g_{ic}(\cdot)$ the first wavelength of each segment (*break point*) c, $m_{ic}(\cdot)$ the slope of the segment, and $h_{ic}(\cdot)$ is the y-intercept value. We model both $g_{ic}(\cdot)$ and the XPM value for each break point ($f_{ic}(\cdot)$), and thus $m_{ic}(\cdot)$ and $h_{ic}(\cdot)$ can be computed from $g_{ic}(\cdot)$, $g_{i(c+1)}(\cdot)$, $f_{ic}(\cdot)$, and $f_{i(c+1)}(\cdot)$.

$$s^2_{XPM}(\alpha_e,\lambda,i) = \begin{cases} m_{ic}(\alpha_e)\cdot\lambda + h_{ic}(\alpha_e), \\ g_{ic}(\alpha_e)\leq\lambda\leq g_{i(c+1)}(\alpha_e), c=1..C \end{cases} \quad (3)$$

Aiming at reducing the number of coefficients for the model ($f_{ic}(\cdot)$ and $g_{ic}(\cdot)$), which are data to be stored, every coefficient can be modeled using mathematical expressions, e.g., polynomials, exponential forms [8]. However, to have a linear function suitable for using in ILP formulations, λ cannot be part of these expressions. After comparing the performance of several alternative linear models, we propose modeling $f_{ic}(\cdot)$ with a polynomial of degree ρ, using α_e as the only variable (x). Eq. (4) illustrates the proposed model for $f_{ic}(\cdot)$, where t_{icj} represents the j-th coefficient of the polynomial:

$$f_{ic}(x) = \sum\nolimits_{j\in[0,\rho]} t_{icj}\cdot x^j \quad (4)$$

Note that $g_{ic}(\cdot)$ represents an integer in the range $[1,|W|]$, where $g_{i1}(\cdot)$=1 and $g_{i(C+1)}(\cdot) = |W|$. To avoid rounding operations, which result in a non-linear expression, Eq. (5) defines a linear function that predicts integer $g_{ic}(\cdot)$ values.

$$g_{ic}(x) = b_{ic}\cdot x + a_{ic}|a_{ic}, b_{ic}\in Z^+ \quad (5)$$

Note that for each segment defined in (3), $f_{ic}(\cdot)$ needs $(\rho+1)$ coefficients to be modeled, while $g_{ic}(\cdot)$ needs two. As a result the total size of the restricted linear model is $2\eta\cdot(C\cdot(3+\rho))$. A good value for ρ and η is a tradeoff between the need to obtain the best goodness-of-fit while keeping the size of the model at a minimum. To this end, a two-step statistic approach was used. First the number of segments C was optimized and then ρ, and consequently t_{icj}, was minimized. The optimal values were obtained applying the well-known least squares minimization fitting [8] over a set of exact $\sigma^2_{XPM}(e,\lambda,i)$ values computed using the equation and reference values presented in [1] (hereafter, *analytical model*). The XPM noise variance was computed for each link with $|W|=80$, assuming a 50 GHz grid, and $\alpha_e\in[1,25]$.

The Pearson determination coefficient (R^2) and a normalized mean squared error (MSE) [8] were used to discriminate among the different models. The normalized MSE was obtained by comparing the MSE of a given model against the MSE of the *null model* which contains only one value representing the average of all the $\sigma^2_{XPM}(e,\lambda,i)$, i.e., every channel in every link produces the same XPM noise regardless of the number of optical amplifiers in the link and its spectral position. Following the above methodology, we first generated different sets of break points ensuring the integrality condition in (5) for values of $C\in[2,5]$. Applying linear regression among values of each segment, we obtained a valid fit (R^2>99%) for C=4. Since C is set, the next step is to find the minimum value of ρ that fits the required target error (R^2>99%). A polynomial of the form described in (4) was obtained applying polynomial fitting for every subset of slopes $f_{ic}(\cdot)$ of the optimal set of slopes F. Algorithm 1 details the algorithm to compute ρ and t_{icj} coefficients for the polynomials. After running this algorithm, we obtain a R^2 > 99% for all the subsets of slopes when ρ=4.

Algorithm 1 Algorithm to Compute ρ and t_{icj} Coefficients

INPUT: $I = \{i \in W, i \geq max(1, \lambda - \eta), i \leq \min(\lambda + \eta, |W|), C, f_{ic}$ values (set F)
OUTPUT: $T = \{t_{icj}\}, \rho$
$\rho \leftarrow 1, T \leftarrow \emptyset, stop \leftarrow false, minR^2 \leftarrow 0.99$
while not stop **do**
  $stop \leftarrow false$
  for all $i \in W$ **do**
    for $c = 1..C$ **do**
      Compute the ρ-th degree polynomial f for all elements in $F \cap \{i, c\}$
      if $R^2(f) \geq minR^2$ **then**
        $T \leftarrow T \cup \{f$ coeffs.$\}$
      else
        $T \leftarrow \emptyset$, $\rho++, stop \leftarrow false$
        break for
      end if
    end for
    if stop=false **then**
      break for
    end if
  end for
end while

III. Performance Evaluation

The performance of the proposed restricted linear XPM model has been evaluated in terms of accuracy and size. Fig. 2 plots both $\sigma^2_{XPM}(e, \lambda, i)$ exact values (markers) and $s^2_{XPM}(\alpha_e, \lambda, i)$ fitted ones (solid lines) for $i = \lambda + 1$ (Fig. 2a) and $i = \lambda + 2$ (Fig. 2b). For the sake of a broad comparison, we depict three different link lengths in terms of α_e, i.e., 3, 12, and 23. As illustrated, the higher the value of the XPM variance the better the fitted value. These results reveal that the restricted linear model is accurate enough when compared with the analytical one. Regarding the size of the proposed XPM model, assuming $C = \rho = 4$, the number of coefficients to be stored falls to only 224, compared to $2\eta \cdot |W| \cdot \max\{\alpha_e, e \in E\}$ (16,000 for 80 wavelengths) needed using Eq. (2) with pre-computed values.

Fig. 2c illustrates the goodness-of-fit of the statistical Q model when the restricted linear XPM model is used and the worst case is assumed for FWM; dashed lines represent an error of 5%. As observed, all the fitted values (markers) are within the error range. In a deeper analysis, not shown in the figures, we observed that the restricted linear XPM model provides Q values slightly higher than the analytical one, thus sub-estimating XPM which leads to an over-estimation of Q.

Finally, to weight the impact of the error in the statistical Q computation, the model was evaluated in terms of wrong decisions made in accepting/rejecting lightpaths, i.e., on whether or not the Q value of a lightpath is better than a given Q threshold (Q^{th}). With this objective in mind six different thresholds (ranging from 7 to 12) were considered, each one tested with 15,000 randomly generated lightpaths from a wide set of link lengths, $|W|$ values, hop count, and channels in use in each link. For each lightpath the Q value was computed using the analytical and the restricted linear models. A decision was considered as wrong when the result of the two models were different. Table I details the results in percentage as a function of Q^{th}. As shown the percentage of wrong decisions made by the restricted linear model is lower than 2.1%, which represents a very low error. In fact, the on-average error in the wrong decisions is lower than 1.5%.

TABLE I
Statistical Q Model Validation (Wrong Decisions)

Q^{th}	**7**	**8**	**9**	**10**	**11**	**12**
Wrong decisions (%)	0.9	2.1	2.0	1.7	0.9	0.4
Stat. vs. analy. Q error (%)	1.1	1.3	1.3	1.4	1.5	1.3

To reduce even more the number of wrong decisions, the value of Q^{th} could be slightly increased (in the order of 1-1.5%) being thus decisions taken under a bit more stringent threshold. In the light of these results, we can conclude that the proposed linear XPM model provides an accurate statistical Q estimation really close to the exact Q values.

IV. XPM Model in ILP Formulations and Discussion

This section presents an ILP formulation that can be used to solve the off-line IA-RWA problem. The formulation makes use of the restricted linear XPM model defined in Section II. To this end, Eq. (1) needs to be rearranged. As already mentioned in Section I, PLI data pertinent to linear impairments depend only on the length of the route. Hence, they can be pre-computed if the ILP uses an arc-path formulation [9]. Thus, only σ^2_{XPM} and σ^2_{FWM} are dependent on both the route and the wavelength assignment. We assume a worst case for the FWM noise variance ($\sigma^2_{FWM}(r)$), being that a constant value for our problem. Therefore, from (1), Q^{th} can be translated into a XPM threshold (XPM^{th}) for a given feasible route as:

$$XPM^{th}(r) = \left(\frac{pen_{eye} \cdot P_{tx}}{pen_{PMD} \cdot Q^{th}}\right)^2 - \sigma^2_{ASE}(r) - \sigma^2_{FWM}(r) \quad (6)$$

Obviously, a lightpath whose σ^2_{XPM} value is less than XPM^{th}, has a Q higher than Q^{th}. Then, an ILP formulation for the off-line IA-RWA problem will focus on guaranteeing that the σ^2_{XPM} value of each established lightpath (computed using the s^2_{XPM} piece-wise linear functions described in (3)) is lower than XPM^{th}. Note that, as previously stated, the right-hand term in (6) can be calculated beforehand for each pre-computed route r.

Let us consider a set of traffic demands D to be served over a network represented by the graph $G(N, E, W)$. For each demand, a set of routes $R(d)$ is pre-computed, where each route is represented as a set of links $E(r) \subseteq E$. Parameter δ_{re} is equal to 1 if route r uses link e.

The off-line IA-RWA consists in finding a route and assigning a wavelength (lightpath) for every demand provided that the Q-factor of each of them does not violate a given Q^{th}. In our example, the objective function consists in minimizing the number of used wavelength channels. However, any other objective function could be defined. The variables are defined as follows: x_{drw}, binary, equal to 1 if demand d uses route r and wavelength w; 0 otherwise. y_{ew}, binary, equal to 1 if wavelength w in link e is used; 0 otherwise. s_d, real positive with the XPM noise variance of demand d. Finally, the off-line

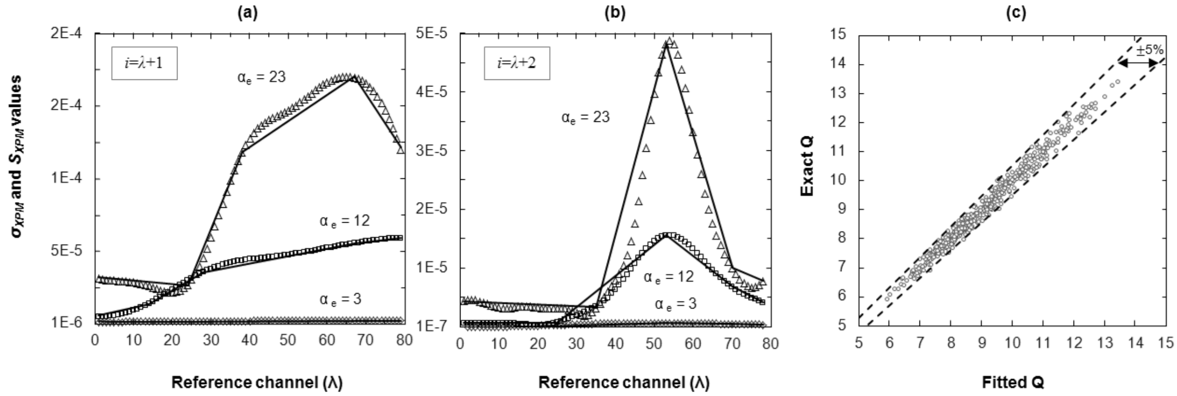

Fig. 2. $\sigma^2_{XPM}(e, \lambda, i)$ and $s^2_{XPM}(\alpha_e, \lambda, i)$ for $i = \lambda + 1$ (a) and $i = \lambda + 2$ (b). Analytical vs. statistical Q values of paths (c).

IA-RWA problem can be modeled as:

$$min \sum_{e \in E} \sum_{w \in W} y_{ew} \tag{7}$$

subject to:

$$\sum_{r \in R(d)} \sum_{w \in W} x_{drw} = 1, \quad \forall d \in D \tag{8}$$

$$\sum_{d \in D} \sum_{r \in R(d)} \delta_{re} \cdot x_{drw} = y_{ew}, \quad \forall e \in E, w \in W \tag{9}$$

$$\sum_{e \in E} \sum_{\substack{w'=w-\eta \\ w' \neq w}}^{w+\eta} \delta_{re} \cdot y_{ew'} \cdot s^2_{XPM}(\alpha_e, w, w') - (1 - x_{drw}) \cdot M \leq s_d, \quad \forall d \in D, r \in R(d), w \in W \tag{10}$$

$$s_d \leq \sum_{r \in R(d)} \sum_{w \in W} x_{drw} \cdot XPM^{th}(r), \quad \forall d \in D \tag{11}$$

The objective function (7) minimizes the total used capacity. Constraint 8 guarantees that each demand is assigned to only one route and wavelength, whereas constraint (9) ensures that each wavelength channel supports only one demand. Constraint (10) computes the XPM noise of each demand according to its route and the occupation of the network, using the restricted linear model for $s^2_{XPM}(\alpha_e, \lambda, i)$. In constraint (10), the use of a big integer M allows the XPM noise to be computed only for those assigned routes and wavelengths. The XPM noise is compared to XPM^{th} in constraint (11) to ensure that all the demands experience a XPM noise lower than the threshold and, consequently, the Q-factor is guaranteed to be higher than the required Q threshold.

As shown, the number of variables and constraints is $O(|D| \cdot Rmax \cdot |W| + |E| \cdot |W|)$, where $Rmax$ is the maximum number of pre-computed routes for each demand. For illustrative purposes, assuming 50 demands, 10 routes for each demand, 16 wavelengths, and the EON-BT topology [6], the size of the problem instance would be lower than 10.000 variables and constraints, which clearly shows the applicability of the proposed XPM model.

To conclude, in this paper a linear statistical model to compute the XPM noise variance for 10 Gbps signals with OOK modulation has been presented. The model allows to accurately estimate the Q factor of lightpaths, taking advantage of the fact that, under the considered scenario, the XPM variance is several times higher than the one of FWM. Exhaustive evaluation confirmed the accuracy of the Q-factor estimation. The XPM model was eventually integrated into an ILP formulation of the off-line IA-RWA problem.

Acknowledgment

The research leading to these results was supported by the MINECO TEC2011-27310 ELASTIC project.

References

[1] A. Jirattigalachote, *et al.*, "ICBR-Diff: an impairment constraint based routing strategy with quality of signal differentiation," *J. Netw.*, vol. 5, pp. 1279–1289, 2010.

[2] S. Pachnicke, *Fiber-Optic Transmission Networks—Efficient design and Dynamic Operation*. Springer, 2011.

[3] K. Manousakis, K. Christodoulopoulos, and E. Varvarigos, "Impairment-aware offline RWA for transparent optical networks," in *Proc. 2009 IEEE INFOCOM*, pp. 1557–1565.

[4] P. Pavon-Mariño *et al.*, "Offline impairment aware RWA algorithms for cross-layer planning of optical networks," *IEEE/OSA J. Lightwave Technol.*, vol. 27, pp. 1763–1775, 2009.

[5] S. Ten *et al.*, "Comparison of four-wave mixing and cross phase modulation penalties in WDM systems," in *Proc. 1999 OFC*.

[6] L. Velasco, *et al.*, "Statistical approach for fast impairment-aware provisioning in dynamic all-optical networks," *IEEE/OSA J. Opt. Commun. Netw.*, vol. 4, pp. 130–141, 2012.

[7] S. Pachnicke and E. Voges, "Analytical assessment of the Q-factor due to cross-phase modulation (XPM) in multispan WDM transmission systems," in *Proc. 2003 SPIE*.

[8] D. Montgomery, *Design and Analysis of Experiments*. Wiley & Sons, 2004.

[9] M. Pióro and D. Medhi, *Routing, Flow, and Capacity Design in Communication and Computer Networks*. Elsevier, 2004.